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Quantifying Uncertainty in Ray-Tracing Models of Radiowave Propagation Using Polynomial Chaos

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***Abstract*—Randomness in the input parameters of a ray-tracing simulator introduces uncertainty in the predictions of the received power and voltage. The polynomial chaos method is applied to efficiently estimate the uncertainty arising from randomness in the material properties for a site-specific ray-tracing analysis of an indoor hallway. The uncertainty is compared against a converged set of Monte-Carlo simulations and with experimental measurements of the sector-averaged received power. Results indicate a 2–3 dB variation in the received power can exist for relatively small material parameter uncertainties.**

***Index Terms*—Ray-Tracing, Uncertainty, Propagation**

I. Introduction

Ray-tracing is widely used to estimate characteristics of indoor and outdoor wireless channels by modelling radio wave propagation using ray-optical approximations [1], [2]. The power and field distributions within complex and detailed environments can found by considering the interactions (e.g. reflection and diffraction) of the rays with each surface or edge in the environmental model [3, pp. 217–251]. Similar to other deterministic methods used to estimate the wireless channel, the accuracy of the ray-tracing predictions when compared to experimental measurements depends on the level of detail included in the environmental model. However, considerable uncertainty can exist in the description of the environment. For example, dimensions on plans and technical drawings are typically specified to be within a certain tolerance. Similarly, material properties such as the dielectric constant and conductivity often vary between samples due to different composition or other effects (e.g. atmospheric moisture content) [4].

The uncertainty in the dimensions and material properties can be expressed as random variables, with the actual dimensions and dielectric properties viewed as a particular realization [5, pp. 1–3]. Other uncertainties, such as randomness in the antenna radiation patterns and random measurement errors can be similarly included. These uncertainties in the inputs will ‘propagate’ through the ray-tracing algorithm to introduce uncertainties and randomness in the results. The size of the output uncertainty will depend how the input uncertainties interact with the ray-tracing model. It should be noted that a single simulation run at the nominal input values (or otherwise) will not provide a measure of the uncertainty in the output, which can only typically be gained by collating multiple results.

Characterizing the uncertainty in the outputs gives an indication of the range of values that are expected, and can give a measure of confidence in the ray-tracing results [6]. Using the results from the uncertainty analysis we can also determine the sensitivity of the model to the various input parameters, and thereby determine which inputs contribute most toward the outputs. This information can be used to help refine the description of the problem, potentially reducing the output uncertainty when compared against experimental measurements [6]. Previous application of ray-tracing analysis to model indoor and urban propagation has shown considerable variability can exist in the results—e.g field strengths and received power—depending in the input parameters, such as the geometrical detail, dimensions and material properties [7]. This paper shows how the variability in the ray-tracer predictions (given the statistics of the input parameters) can be efficiently estimated using a surrogate model based on polynomial chaos expansions.

II. Methods to Estimate Uncertainty

A. Monte Carlo Method

The Monte Carlo method is widely used to estimate statistics and quantify uncertainty in numerical models: a large number of random inputs are generated from the probability distributions of the input parameters, and the model is solved/simulated for each realization. The statistics and uncertainty are then estimated by collating the random solutions. The Monte Carlo method is relatively easy to apply and is used in commercial ray-tracing packages, e.g. Wireless InSite [8]. However, the slow convergence rate of the Monte Carlo method tends to limit its application to computationally large problems [9].

B. Polynomial Chaos Method

As shown in Fig. 1, the polynomial chaos method is used to construct a surrogate model for the ray-tracer results, R, in terms of the system inputs. In particular, R' is approximated by finite summation of weighted orthogonal polynomial basis functions in the input parameter space. The polynomial chaos method converges significantly faster than the Monte Carlo method and generally requires fewer runs of the full model to construct the surrogate model. Similar to the Monte Carlo method, the polynomial chaos expansion and resulting surrogate model is specific to the problem and geometry under investigation. Statistics computed from the surrogate model will be a good approximation to the statistics of the

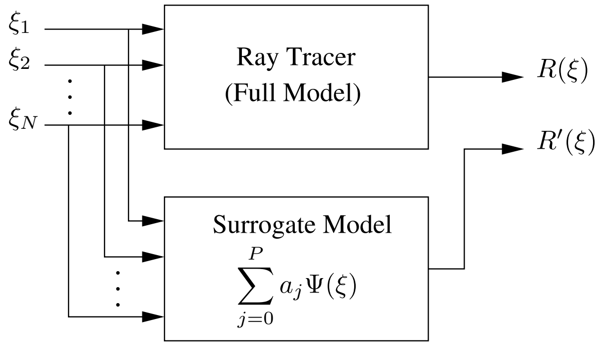

Fig. 1. The polynomial chaos expansion is used to derive the surrogate model for the ray-tracer. A small number of full model simulations are required to form the expansion. Uncertainty in R' can be computed directly from the expansion coefficients.

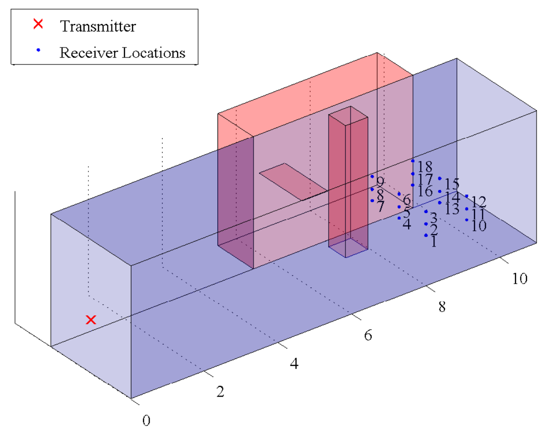

Fig. 2. Depiction of the indoor hallway considered in this analysis. The permittivity of the side walls, floor and ceiling are assumed to be independent and follow Gaussian distributions. To eliminate multi-path fading, the received power is averaged over 49 points in the vicinity each of the 18 receiver locations identified. Further research will investigate the impact of other environmental details indicated in red.

full system. Furthermore, the surrogate model can be rapidly evaluated for any combination of input parameters as it only contains polynomial terms.

The polynomial chaos expansion for N random inputs $\boldsymbol{\xi} = \{\xi_1, \ldots \xi_N\}$ is given by

$$R(\boldsymbol{\xi}) \approx R'(\boldsymbol{\xi}) = \sum_{j=0}^{P} a_j \Psi_j(\boldsymbol{\xi}) \tag{1}$$

where a_j is the weighting coefficient for the multivariate polynomial chaos basis $\Psi_j(\cdot)$ [10, pp. 57–67]. The number of terms P is given by $\frac{(N+D)!}{N!D!}$, where D is the highest polynomial order. It can be shown that the optimal polynomial basis, $\Psi(\boldsymbol{\xi})$, depends on the probability distribution of the random variables, $\boldsymbol{\xi}$ [9]. For example, Hermite polynomials are associated with Gaussian variates and Legendre polynomials with Uniform variates.

The weighting coefficients for the surrogate model can be found by projection

$$\begin{aligned} a_j &= \frac{\langle R(\boldsymbol{\xi}), \Psi_j(\boldsymbol{\xi})\rangle}{\langle \Psi_j^2(\boldsymbol{\xi})\rangle} \\ &= \frac{1}{\langle \Psi_j^2(\boldsymbol{\xi})\rangle} \int_{\Omega^N} R(\boldsymbol{\xi})\, \Psi_j(\boldsymbol{\xi})\, \mathrm{d}\boldsymbol{\xi} \end{aligned} \tag{2}$$

where the integration is over the N-dimensional input parameter space, Ω^N. The multi-dimensional integral in (2) can be evaluated using numerical quadrature, e.g.

$$\int_{\Omega^N} R(\boldsymbol{\xi})\, \Psi_j(\boldsymbol{\xi})\, \mathrm{d}\boldsymbol{\xi} \approx \sum_q R\left(\boldsymbol{\xi}^{\{q\}}\right) \Psi_j\left(\boldsymbol{\xi}^{\{q\}}\right) w^{\{q\}} \tag{3}$$

where $\boldsymbol{\xi}^{\{q\}}$ and $w^{\{q\}}$ are the quadrature points and weights respectively. Thus to construct the surrogate model, the full model (i.e. the ray-tracer) needs to be evaluated for a set of $\{q\}$ input parameters, corresponding to the quadrature points in (3). Sparse grid integration techniques, such as the Smolyak algorithm [11], and nested quadrature rules (e.g. Kronrod-Patterson) are used to reduce the number of quadrature points required [12].

III. Application to Ray-Tracing

The polynomial chaos method outlined in the previous section is applied to an image-based ray-tracing analysis [2] of an indoor hallway at 2.28 GHz shown in Fig. 2. Red shaded objects in the environment were excluded from this analysis but will be included in further research. The ray-tracing results are compared against a site-survey of the channel: the transmitter is located at point $\times$, and to remove the effects of multi-path fading, the received signal at each of the 18 receiver locations is averaged over a grid of 49 points using a linear x–y translator. The transmitting antenna is a quarter-wavelength monopole above a ground plane, while at the receiver end of the system a high efficiency meta-material antenna is used [13]. The three dimensional radiation patterns of both antennas were embedded in the ray-tracer. For this analysis, the maximum number of ray interactions was restricted to six, which provided an acceptable trade-off between accuracy and speed.

The walls of the hallway are constructed from drywall, while the floor and ceiling are formed from reinforced concrete. The surfaces are assumed to be electrically smooth and are parameterized in the ray-tracer by their permittivity and conductivity. For this uncertainty analysis the permittivity of the drywall is assumed to follow a Gaussian distribution, with mean 3.0 and standard deviation 0.5; while the permittivity of the floor and ceiling are assumed to follow Gaussian distributions with mean 6.0 and standard deviation 1.0. The permittivity of the walls, floor and ceiling are further assumed to be statistically independent. Previous analysis [6] has shown conductivity does not have a significant effect for strongly reflected signals, thus for this analysis the conductivity for all materials is assumed to be 1 mS/m.

Fig. 3 shows the 90% confidence interval (CI) for the sector-

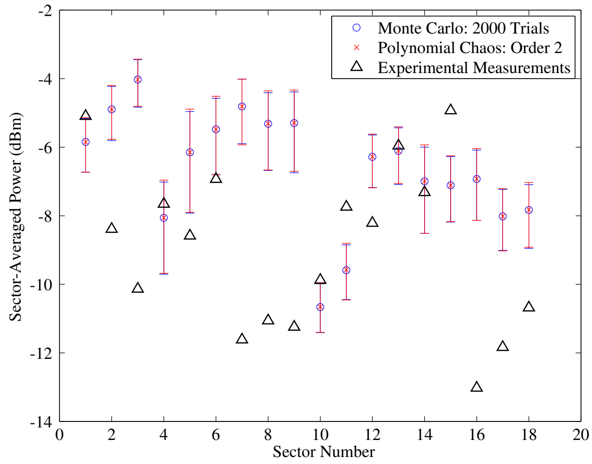

Fig. 3. Uncertainty in the sector-averaged received power (in dB units) due to randomness in the permittivity. While good agreement is observed between the Polynomial Chaos and Monte-Carlo results, the deviation from the experimental measurements suggests a more detailed environmental model is needed.

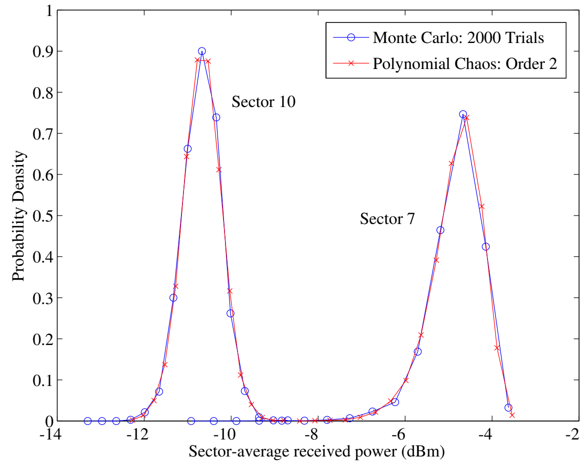

Fig. 4. Estimated probability density functions of the sector-averaged power (in dB units) for sectors 7 and 10.

averaged received power computed at each of the 18 sectors given the uncertainties in the material properties. The 90% CI gives a measure of the expected range about the predicted mean. For most sectors, uncertainty in the material properties introduces 2–3 dB variation about the mean sector-averaged power. The polynomial chaos surrogate model (truncated at $D = 2$) is constructed from 19 runs of the full ray-tracing simulator, by contrast 2000 Monte Carlo simulations were required—this represents an approximately 100-fold decrease in the computational requirements. Many experimental measurement points are observed to fall outside the 90% confidence interval of the simulated results. This suggests the differences between the ray-tracer and measurements cannot be entirely attributed to uncertainty in the material parameters. A refined geometrical model incorporating further features of the environment, such as concrete pillars and alcoves is currently under consideration to improve the accuracy of the ray-tracer.

Fig. 4 shows probability density functions (PDFs) of the sector-averaged power in dB units for sectors 7 and 10 computed using 2000 Monte-Carlo trials and polynomial chaos expansions truncated at $D = 2$. For both sectors, statistics (mean and standard deviation) computed using the polynomial chaos results compare well with the Monte-Carlo simulations, and similar observations can be made for the other sectors.

IV. Summary

Deterministic methods used to model the radio channel, such as ray-tracing, require a detailed description of the problem geometry that includes all relevant dimensions and materials (with the associated dielectric properties). Inherent randomness in the description of the environment introduces uncertainty in ray-tracing predictions of the radio channel. Results from the ray-tracing analysis of a hallway environment show the received power can vary significantly due to uncertainties in the material properties. Efficiently quantifying these uncertainties is important to assess the sensitivity of the predictions and can give a measure of confidence in the simulated results. Polynomial chaos based surrogate modelling allows the statistics and uncertainty to be examined at significantly lower computational cost than competing methods. Further analysis of the uncertainty due to position are currently being examined.

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