Department of Electrical and Computer Engineering

ELECTENG 721 Radio Systems
ELECTENG 737 Advanced Radio Systems

Receiver Design

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1 The receiver front-end

In a typical radio system the received signal can be very weak (due to propagation loss). A high gain amplifier is thus required to increase the signal so that it will fall in the range that can be captured with the detector. Fig. 1 shows the superheterodyne receiver configuration, which forms the basis of most modern receiver designs. The variable frequency **Local Oscillator** is used to tune the receiver so that the desired frequency in the incoming signal is down-converted by the mixer to a (fixed) **Intermediate Frequency (IF)**. The advantage of the superheterodyne configuration is that all components (e.g., filters) from the IF stage onward can be standardised. It is difficult to make *tunable* and *selective* filters for RF frequencies.

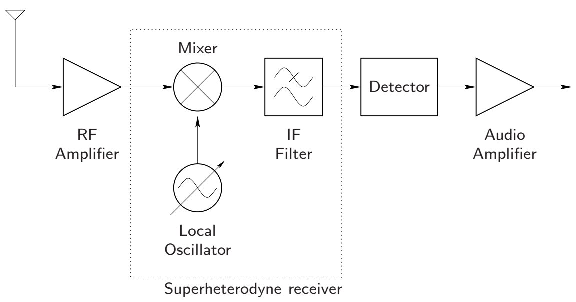

Fig. 1: Block diagram for a superheterodyne receiver.

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2 Linearity

Components of a radio system that (generally) are *linear* include:

- Antennas;
- Transmission lines; and
- Matching networks and passive RF filters.

A key property of linear systems is superposition, e.g., suppose the input signal, x, to a linear system $G(\cdot)$ consists of two components a and b, i.e., $x = a + b$ then the output y, is $y = G(a) + G(b)$. This property is useful as it allows us to decompose a signal and analyse the system response to each component separately without losing any information. It also implies that no *new* frequency components (that were not already present in x) will be generated.

Components of a radio system that are *non-linear* include:

- Amplifiers;
- Mixers; and
- Circulators.

In this case, we will find that frequency components that were *not* in the input may be present in the output. Sometimes the non-linear behaviour can be beneficial and sometimes detrimental to the performance.

2.1 Taylor series expansion

The output of any non-linear system can be expressed using the Taylor series expansion, given by

$$V_o = a_0 + a_1 V_s + a_2 V_s^2 + a_3 V_s^3 + \dots \tag{1}$$

where V_s is the input signal, V_o is the output signal, and a_n is the coefficient of the n-th term in the expansion. We will consider two types of input signal, *single-tone*:

$$V_s = V \cos \omega t \tag{2}$$

and *two-tone*

$$V_s = V_1 \cos \omega_1 t + V_2 \cos \omega_2 t. \tag{3}$$

We will now analyse each term in the Taylor series expansion (up to third order).

3 Single-tone input

Substituting (2) into (1) leads to

$$V_o = a_0 + a_1 V \cos \omega t + (a_2 V \cos \omega t)^2 + (a_3 V \cos \omega t)^3 + \dots \tag{4}$$

which can be simplified via trigonometric identities to

$$V_o = a_0 + a_1 V \cos\omega t + \frac{a_2 V^2}{2} + \frac{a_2 V^2}{2}\cos 2\omega t + \frac{3a_3 V^3}{4}\cos\omega t + \frac{a_3 V^3}{4}\cos 3\omega t + \dots \tag{5}$$

We observe this expression contains:

0. The DC terms: a_0 and $\frac{a_2 V^2}{2}$, representing the bias/offset conditions;
1. The **linear** terms at ω: $a_1 V \cos\omega t$ and $\frac{3a_3V^3}{4}\cos\omega t$), representing the *gain*—ideally for an amplifier the first term should dominate;
2. A term at 2ω: $\frac{a_2V^2}{2}\cos 2\omega t$, representing **second harmonic distortion**; and
3. A term at 3ω: $\frac{a_2V^2}{2}\cos 2\omega t$, representing **third harmonic distortion**.

Usually the DC and harmonic terms are not a problem as we can filter these off, but sometimes this is not possible, e.g., very large bandwidth systems. Sometimes the higher harmonics may be useful, e.g., a frequency multiplier.

4 Two-tone input

For the two-tone input we start by substituting (3) into (1)

$$V_o = a_0 + a_1 \left(V_1 \cos\omega_1 t + V_2 \cos\omega_2 t\right) + a_2 \left(V_1 \cos\omega_1 t + V_2 \cos\omega_2 t\right)^2 + a_3 \left(V_1 \cos\omega_1 t + V_2 \cos\omega_2 t\right)^3 + \dots \tag{6}$$

We will now consider the second and third order terms in (6) separately.

4.1 $a_2 V_s^2$ term

We can expand the $a_2 V_s^2$ term, leading to

$$\begin{aligned} V_o &= a_2 \left(V_1 \cos\omega_1 t + V_2 \cos\omega_2 t\right)^2 \\ &= a_2 \left(V_1^2 \cos^2\omega_1 t + V_2^2 \cos^2\omega_2 t + 2V_1V_2\cos\omega_1 t\cos\omega_2 t\right). \end{aligned} \tag{7}$$

Similar to (5) the first two terms in (7) give us a DC term and components at $2\omega_1$ and $2\omega_2$. The third term in (7) can be expanded using the product-to-sum identity,

$$\cos\omega_1 t \cos\omega_2 t = \frac{1}{2}\cos\left(\omega_1 + \omega_2\right) t + \frac{1}{2}\cos\left(\omega_1 - \omega_2\right) t. \tag{8}$$

We thus observe components at frequencies that are the sum $(\omega_1 + \omega_2)$ and difference $(\omega_1 - \omega_2)$of the input frequencies. This behaviour can be productively used in an RF device called a **mixer**. As shown in Fig. 2, the mixer is a three-terminal device (two inputs, one output) than functions as an analog multiplier. Mixers are used in RF transmitter circuits to modulate the carrier signal, and similarly on the receiver side to move the received signals to a specified intermediate frequency.

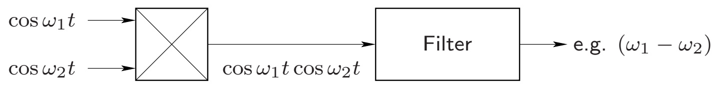

Fig. 2: RF mixer block diagram.

4.2 $a_3 V_s^3$ term

The third order terms are often the greatest culprit in causing **unwanted distortion products**,

$$\begin{aligned} V_o &= a_3 \left(V_1 \cos\omega_1 t + V_2 \cos\omega_2 t\right)^3 \\ &= a_3 \left(V_1^3 \cos^3\omega_1 t + V_2^3 \cos^3\omega_2 t + 3V_1^2 V_2 \cos^2\omega_1 t \cos\omega_2 t + 3V_1 V_2^2 \cos\omega_1 t \cos^2\omega_2 t\right). \end{aligned} \tag{9}$$

The first two terms reduce to the frequency components already discussed, i.e., $\cos^3\omega_1 t \to \omega_1,\ 3\omega_1$ and $\cos^3\omega_2 t \to \omega_2,\ 3\omega_2$. Applying trigonometric identities to the last two terms leads to:

$$\begin{aligned} &\frac{3a_3 V_1^2 V_2}{2}\left(\cos\omega_2 t + \underbrace{\frac{1}{2}\cos\left(2\omega_1 - \omega_2\right)t + \frac{1}{2}\cos\left(2\omega_1 + \omega_2\right)t}_{\text{IMD}}\right) + \\ &\frac{3a_3 V_1 V_2^2}{2}\left(\cos\omega_1 t + \underbrace{\frac{1}{2}\cos\left(2\omega_2 - \omega_1\right)t + \frac{1}{2}\cos\left(2\omega_2 + \omega_1\right)t}_{\text{IMD}}\right). \end{aligned} \tag{10}$$

In addition to the components at ω_1 and ω_2 (these are distortion since the amplitude depends on V_1 *and* $V - 2$), we now have third order **intermodulation distortion (IMD)** components. These appear at frequencies

$$\begin{aligned} &2\omega_1 - \omega_2 \\ &2\omega_1 + \omega_2 \\ &2\omega_2 - \omega_1 \\ &2\omega_2 + \omega_1. \end{aligned}$$

Why are these a problem?

- For example, consider an amplifier with inputs $f_1 = 100$ MHz and $f_2 = 101$ MHz, the ideal (linear) output would be just these two components. The actual output (up to third order terms) will contain components at:
  - 200, 300, 202, 303 MHz (harmonics)
  - 1, 201 MHz (2nd order IMD)
  - 99, 102, 301, 302 MHz (3rd order IMD).

  The *actual* output is sketched in Fig. 3. Some of these components could be filtered, but the components at 99 MHz ($2f_1 - f_2$) and

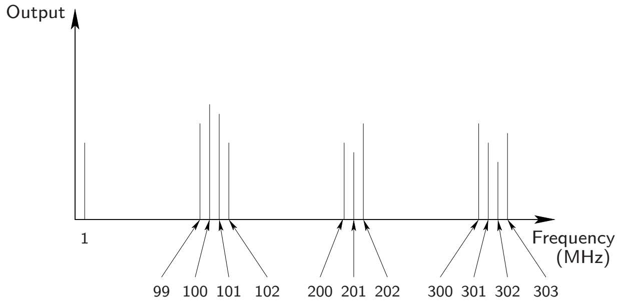

Fig. 3: Output spectrum from a two-tone signal applied to a 3rd order non-linear system

102 MHz ($2f_2 - f_1$) are of particular concern as these are very close to the original transmission frequencies and are very difficult (if not impossible) to filter.

- Consider an input signal to a (non-linear) amplifier with a desired component at 100 MHz, and undesired interfering components at 105 MHz and 110 MHz. The undesired components will produce an IMD component at 100 MHz ($2 \times 105 - 110$). This IMD component is at the same frequency of the desired signal. No amount of filtering can remove this product!

5 Characterising non-linear behaviour

5.1 Gain compression

For an ideal linear amplifier, a plot of the output power (dBm) versus the input power (dBm) would give a straight line, as depicted in Fig. 4. In reality, as the input power increases, a point is reached where the output no longer increases at the same rate—this occurs due to gain compression and results in signal distortion and clipping of the output waveform.

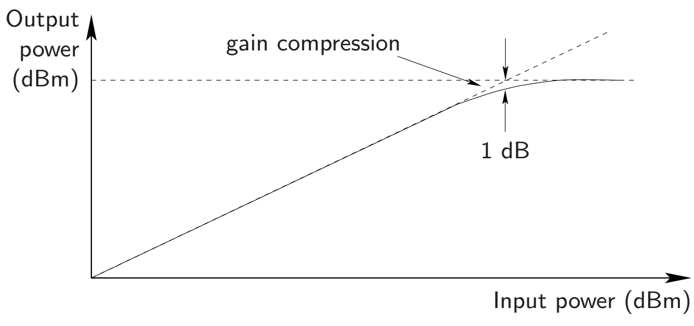

Fig. 4: Gain compression for an amplifier.

For the case where only one signal is present, the output (at ω and

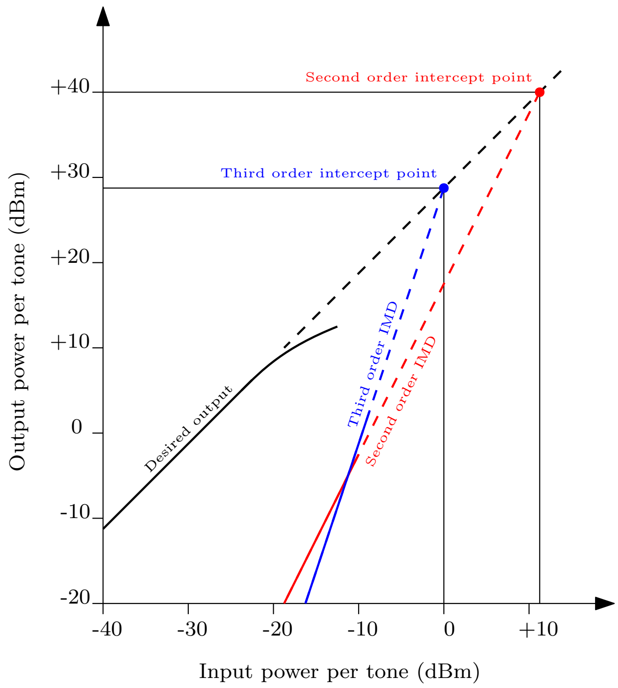

Fig. 5: Second and third order intercept points for an RF amplifier (what is the gain of this amplifier?)

assuming we have sufficiently filtered the harmonics) is

$$V_o = a_1 V \cos \omega t + \frac{3}{4} a_3 V^3 \cos \omega t. \tag{11}$$

The ratio of the gain with distortion to the ideal linear gain is thus

$$\frac{a_1 + \frac{3}{4} a_3 V^3}{a_1} \tag{12}$$

and is referred to as the **single-tone gain compression factor**.

5.2 Intercept points

From (10) and (7) the output amplitude of the n-th order IMD terms is proportional of the input signals raised to the n-th power. Therefore, a plot of the output signal versus the input signal on a log-log scale (i.e., dB-dB), will be straight lines with gradients of 2 and 3 for the second and third order IMD products respectively. These lines will intersect with the linear (desired) output curve at the second and third order intercept points, as shown in Fig. 5.

The intercept points be specified by either the input (e.g., for mixers) or the output (e.g., for amplifiers) power at the intersection, and is a measure of the intermodulation distortion. It should be noted that the component or device would normally be operated well away from the intercept point, is there better parameter to use?

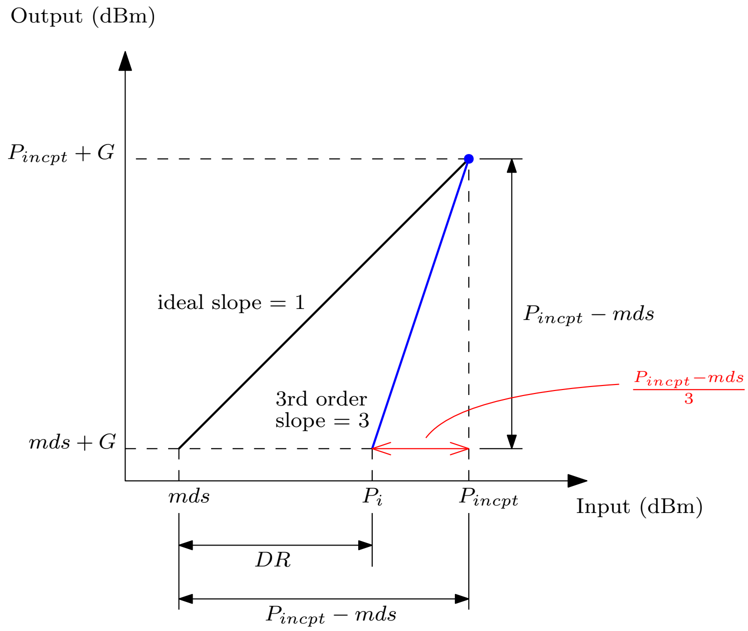

Fig. 6: Receiver dynamic range for a third order IMD terms.

5.3 Two-tone dynamic range

One suitable operating region is where the IMD terms are below the minimum discernible signal (mds) level (i.e., the receiver noise floor). The **dynamic range** of the receiver is defined as the region between the mds (for the linear component) and where the third order IMD terms rise above the mds. The dynamic range for third-order IMD is

$$DR = \frac{2}{3}\left(P_{incpt} - mds\right) \tag{13}$$

which is derived graphically in Fig. 6.

6 Discussion

Non-linear distortion can also be introduced by hardware impairments in the transmitter. Of particular concern are out-of-band emissions, which may cause interference to other neighbouring systems (or users in a multiple-access system).

We have only considered tone inputs, but it is important to recognise that the effects of non-linear distortion only get more complicated for the wideband signals (e.g., 1–100+ MHz bandwidth) typically encountered in modern wireless communication systems. Designing digital and RF systems to compensate for the non-linear distortion introduced by the RF amplifiers (and other non-linear components) remains a challenge.