WE4D-1

Efficient Analysis of Parameter Uncertainty in FDTD Models of Microwave Circuits using Polynomial Chaos

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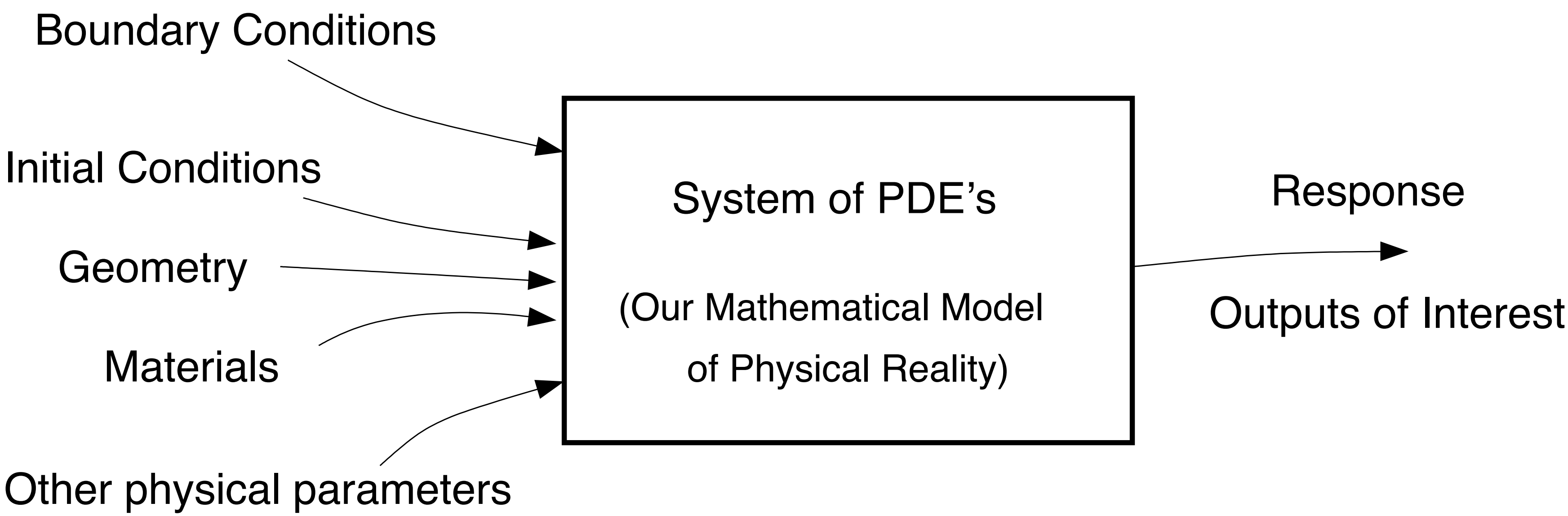
Outline

- Motivations
- Modeling Uncertainty in the FDTD
  - Polynomial Chaos Expansion
  - Uncertainty in dimensions and geometry
- Application and Validation
- Summary

Motivations



- We often do not have complete knowledge of the inputs: randomness / uncertainties exist…
- Uncertainties 'propagate' through the system
- Goal: estimate the uncertainty in the outputs

Polynomial Chaos Expansion

- Possible to expand a 2^{nd} order random process in an orthogonal basis[1]

$$U(\mathbf{x}, t; \boldsymbol{\xi}) \approx \sum_{a=0}^{P} u_a(\mathbf{x}, t)\Psi_a(\boldsymbol{\xi})$$

- $\boldsymbol{\xi} = \{\xi_1, \xi_2, \ldots \xi_N\}$ are the N random inputs and Ψ_a are the polynomial chaos basis functions

- $P + 1 = \frac{(N + D)!}{N!D!}$ D is the highest polynomial order

[1]D. Xiu and Karniadakis, SIAM J. Sci. Comput., **24**(2) 2002

Polynomial Chaos Expansion

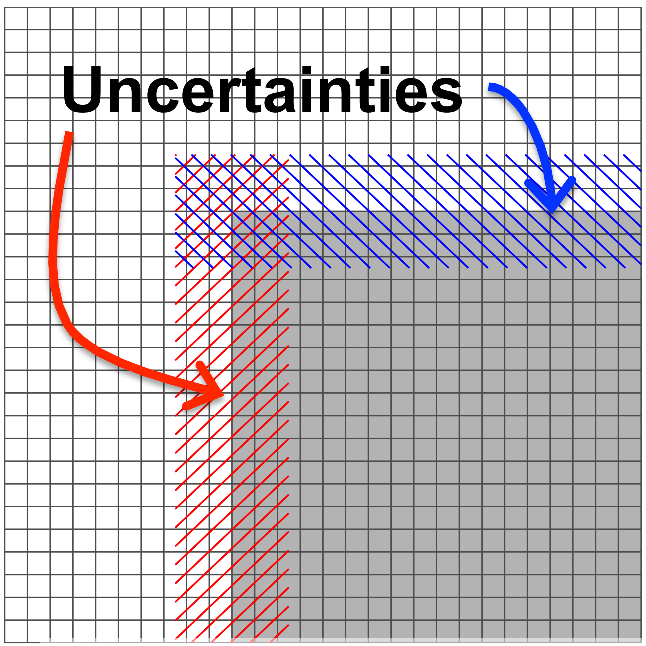
- Basic idea: expand the system's governing equations using PC basis functions
  - Larger, coupled system: harder to solve !

- Advantages: rigorous theory, all statistics can be computed from one simulation
- Previously applied to some CEM problems
  - Material properties (FDTD)[1,2]
  - Coupling to transmission lines[3]

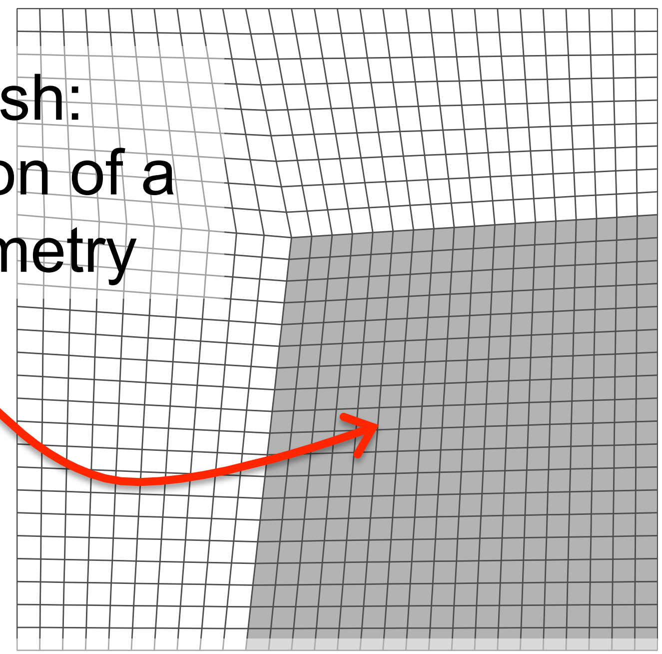
[1]R. Edwards *et al.,* IEEE Trans. EMC-52(1), pp. 155-163, 2010
[2]C. Chauviere *et al.*, SIAM J. Sci. Compat. 28(2) pp. 751-775, 2006
[3]P. Manfredi and F. Canavero, IEEE Trans. EMC-54(3) pp. 677-680, 2012

Uncertain Geometry & Dimensions

- Important for analyzing fabrication tolerances
  - Need to model uncertainty in metal/PEC (previous approaches cannot)



Distorted mesh:
one realization of a random geometry



- Incorporate randomness in the geometry by introducing uncertainty in the FDTD computational mesh

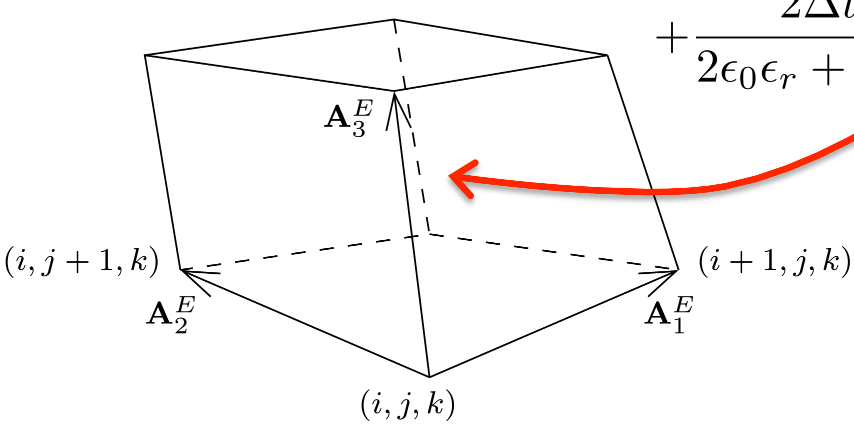
PCE-FDTD Derivation I

- Apply the PCE to expand the field components

$$E^1(i,j,k,n,\boldsymbol{\xi}) = \sum_{a=0}^{P} {}_a e^1\big|_{i,j,k}^{n} \Psi_a(\boldsymbol{\xi})$$

- Substitute into curvilinear FDTD update eqns:

$$\sum_{a=0}^{P} {}_a e^1\big|_{i,j,k}^{n+1} \Psi_a(\boldsymbol{\xi}) = \frac{2\epsilon_0\epsilon_r - \sigma\Delta t}{2\epsilon_0\epsilon_r + \sigma\Delta t} \sum_{a=0}^{P} (\boldsymbol{\xi})\big|_{i,j,k} \, {}_a e^1\big|_{i,j,k}^{n} \Psi_a(\boldsymbol{\xi})$$
$$+ \frac{2\Delta t}{2\epsilon_0\epsilon_r + \sigma\Delta t} \left[\frac{1}{V(\boldsymbol{\xi})\big|_{i,j,k}^{E^1}} \sum_{a=0}^{P} \left({}_a h_3\big|_{i,j,k}^{n+\frac{1}{2}} - {}_a h_3\big|_{i,j-1,k}^{n+\frac{1}{2}} \right) \Psi_a(\boldsymbol{\xi}) - \sum_{a=0}^{P} \left({}_a h_2\big|_{i,j,k}^{n+\frac{1}{2}} - {}_a h_2\big|_{i,j,k-1}^{n+\frac{1}{2}} \right) \Psi_a(\boldsymbol{\xi}) \right]$$



PCE-FDTD Derivation II

- Apply a Galerkin procedure with a test function $\Psi_b(\xi),\ b = 0, \ldots P$

$$
{}_b e^1\Big|_{i,j,k}^{n+1} = \frac{2\epsilon_0\epsilon_r - \sigma\Delta t}{2\epsilon_0\epsilon_r + \sigma\Delta t}\,{}_b e^1\Big|_{i,j,k}^{n} + \frac{2\Delta t}{2\epsilon_0\epsilon_r + \sigma\Delta t}
\cdot \frac{1}{\langle \Psi_b^2 \rangle} \sum_{a=0}^{P} \Big({}_a h_3\Big|_{i,j,k}^{n+\frac{1}{2}} - {}_a h_3\Big|_{i,j-1,k}^{n+\frac{1}{2}} - {}_a h_2\Big|_{i,j,k}^{n+\frac{1}{2}}
+ {}_a h_2\Big|_{i,j,k-1}^{n+\frac{1}{2}} \Big) \left\langle \frac{1}{V(\xi)\Big|_{i,j,k}^{E^1}} \Psi_a(\xi)\, \Psi_b(\xi) \right\rangle .
$$

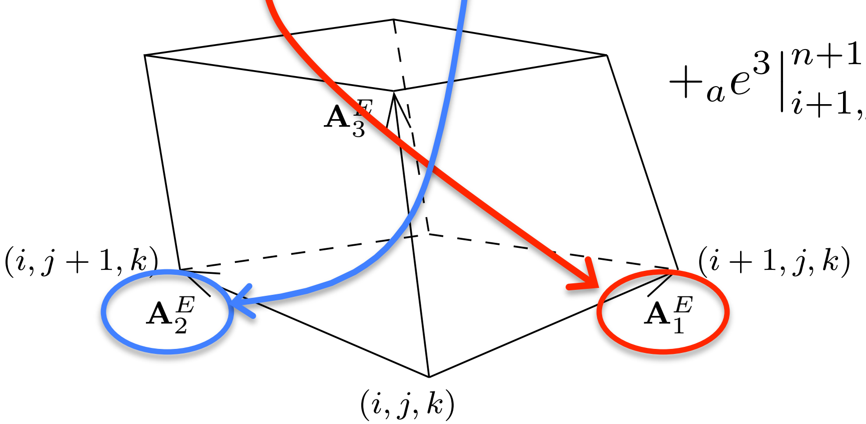
- Inner products can be precomputed using numerical quadrature

PCE-FDTD Derivation III

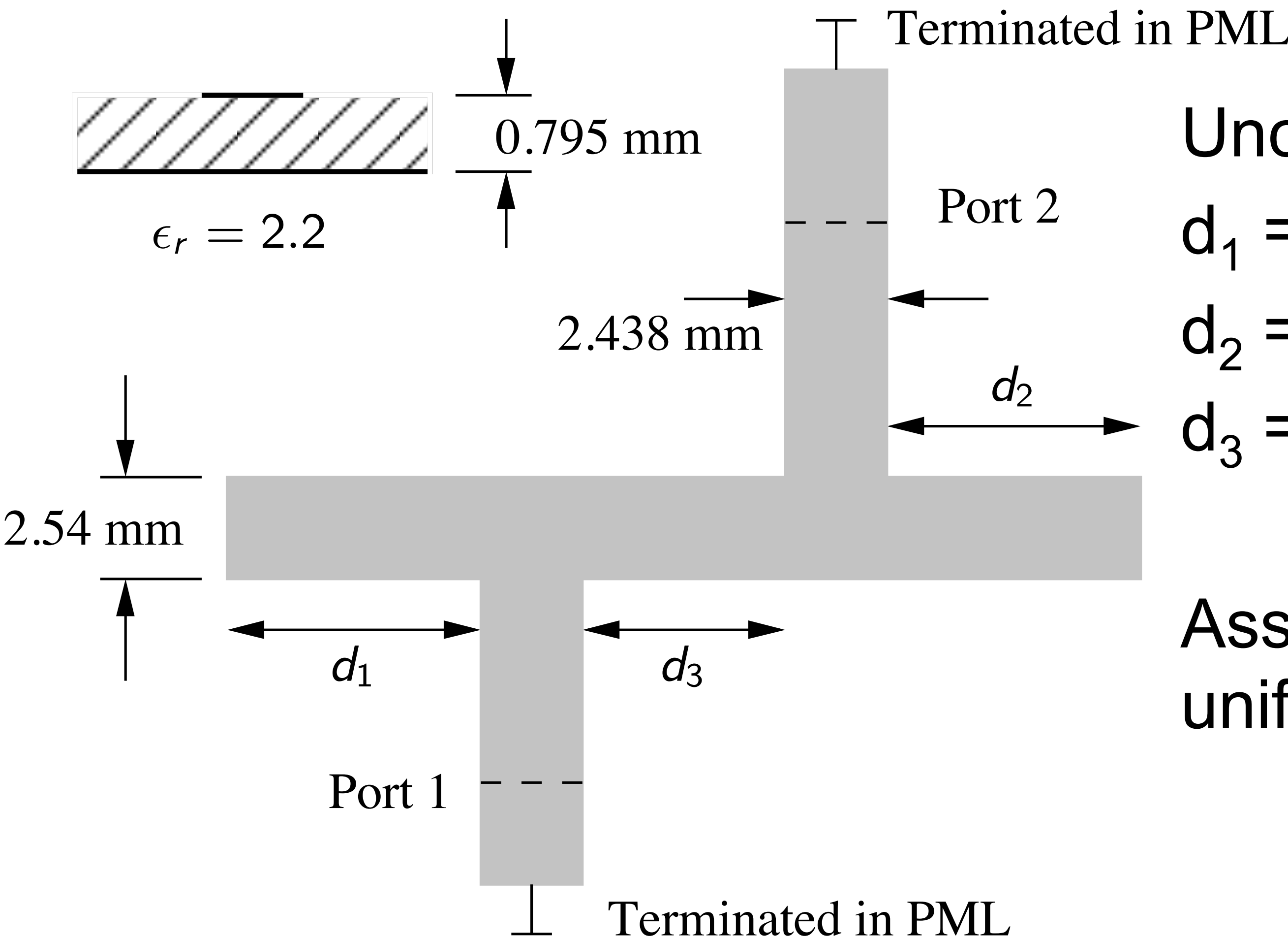
- Need to project contravariant fields:
  - *g*-metrics are uncertain

$$
g_{n,m}^{E}\Big|_{i,j,k} = \mathbf{A}_n^E\Big|_{i,j,k} \cdot \mathbf{A}_m^E\Big|_{i,j,k}
$$

$$
\begin{aligned}
{}_b e_1\Big|_{i,j,k}^{n+1} = \frac{1}{\langle \Psi_b^2 \rangle} \sum_{a=0}^{P} {}_a e^1\Big|_{i,j,k}^{n+1} \left\langle g_{11}^E(\boldsymbol{\xi})\Big|_{i,j,k} \Psi_a(\boldsymbol{\xi})\, \Psi_b(\boldsymbol{\xi}) \right\rangle \\
+ \left[{}_a e^2\Big|_{i,j,k}^{n+1} + {}_a e^2\Big|_{i,j-1,k}^{n+1} + {}_a e^2\Big|_{i+1,j,k}^{n+1} + {}_a e^2\Big|_{i+1,j-1,k}^{n+1} \right] \\
\cdot \left\langle \boxed{\frac{g_{12}^E(\boldsymbol{\xi})\Big|_{i,j,k}}{4}} \Psi_a(\boldsymbol{\xi})\, \Psi_b(\boldsymbol{\xi}) \right\rangle + \left[{}_a e^3\Big|_{i,j,k}^{n+1} + {}_a e^3\Big|_{i+1,j,k}^{n+1} \right. \\
\left. + {}_a e^3\Big|_{i+1,j,k-1}^{n+1} + {}_a e^3\Big|_{i,j,k-1}^{n+1} \right] \left\langle \frac{g_{13}^E(\boldsymbol{\xi})\Big|_{i,j,k}}{4} \Psi_a(\boldsymbol{\xi})\, \Psi_b(\boldsymbol{\xi}) \right\rangle
\end{aligned}
$$



Microstrip Lowpass Filter



Uncertainties:

$d_1 = 5.65 \pm 0.5$ mm

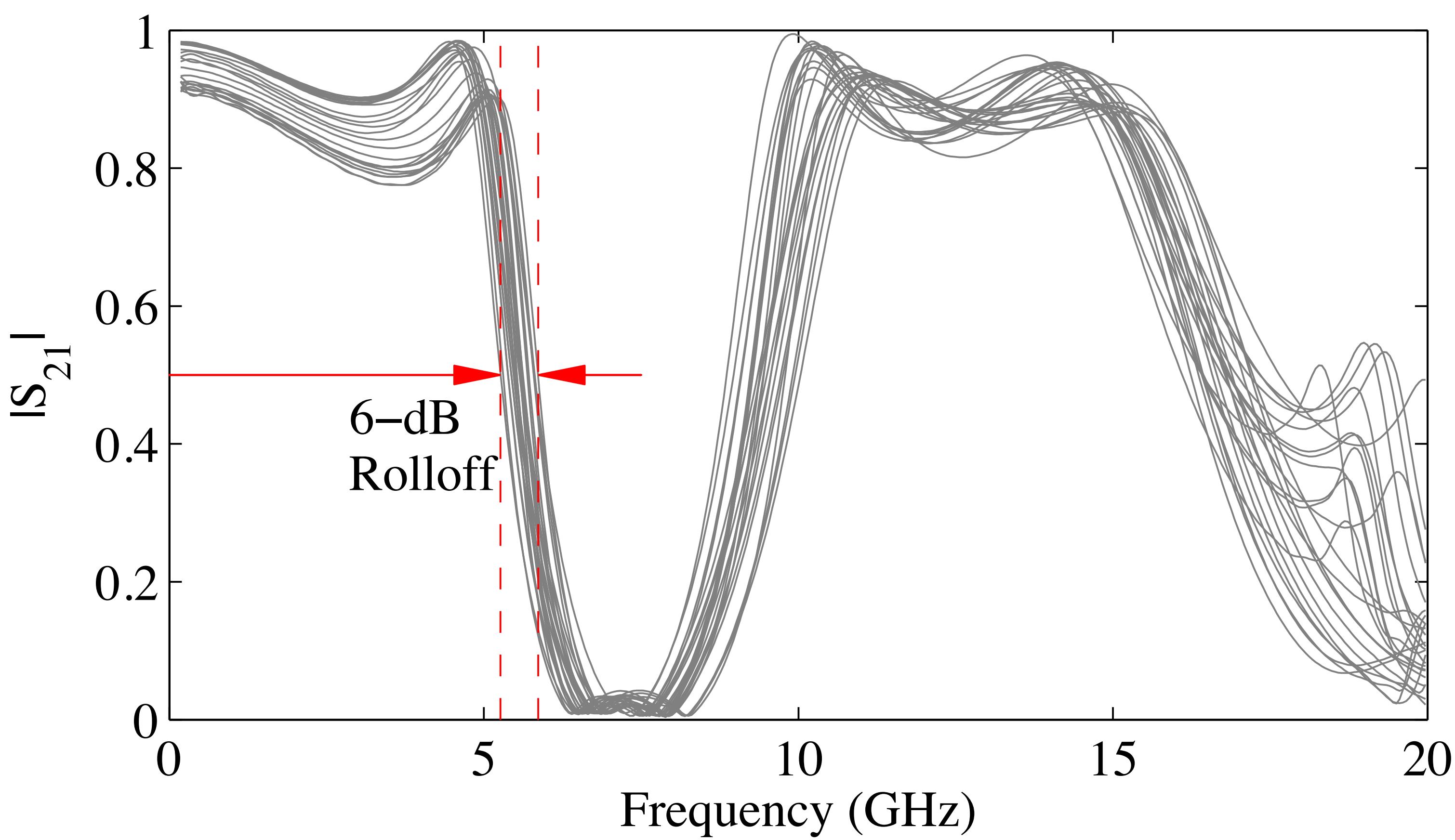
$d_2 = 5.65 \pm 0.5$ mm

$d_3 = 4.06 \pm 0.5$ mm

Assumed to be uniformly distributed

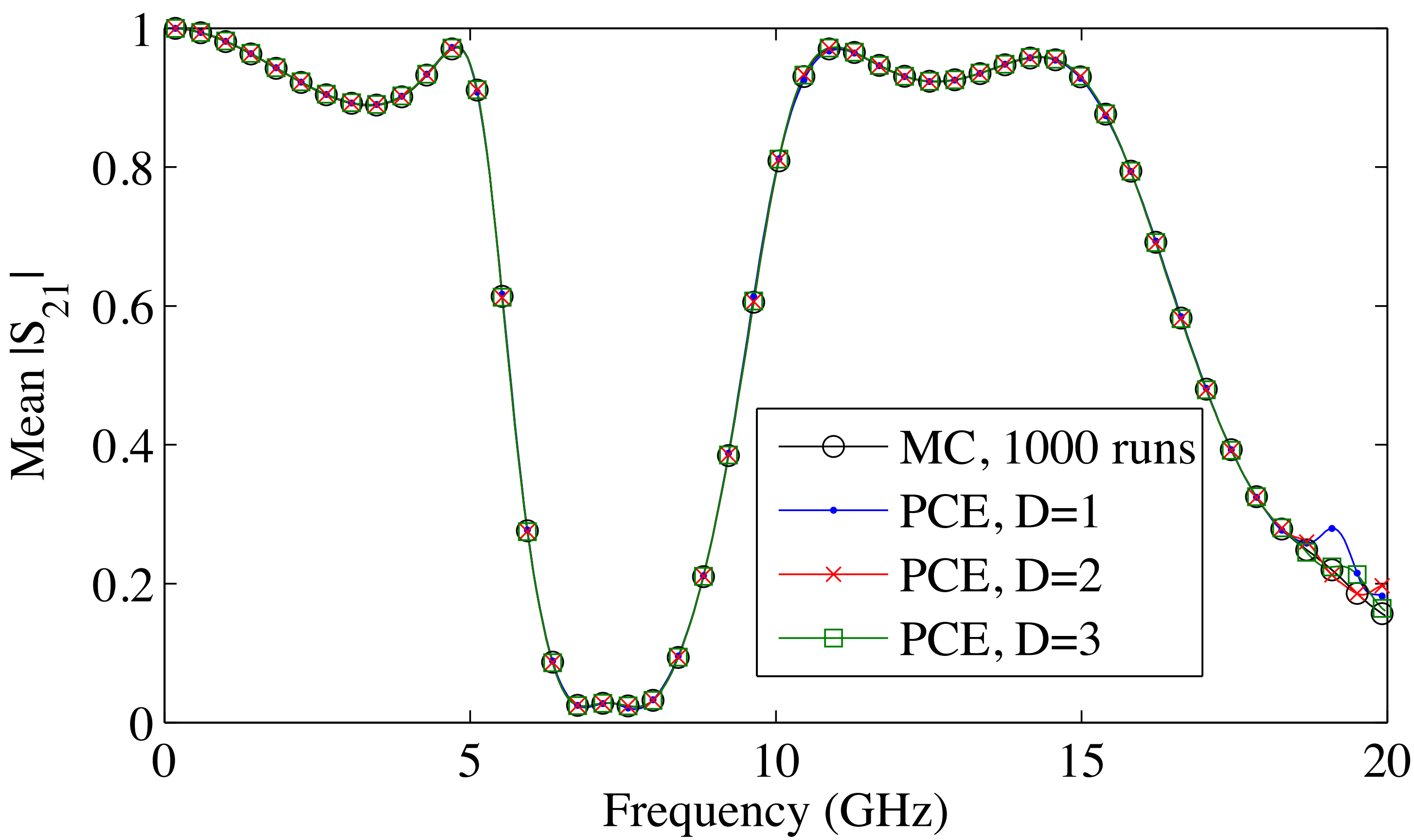
Microstrip Lowpass Filter

- Uncertainty in stubs ➔ randomness in S_{21}



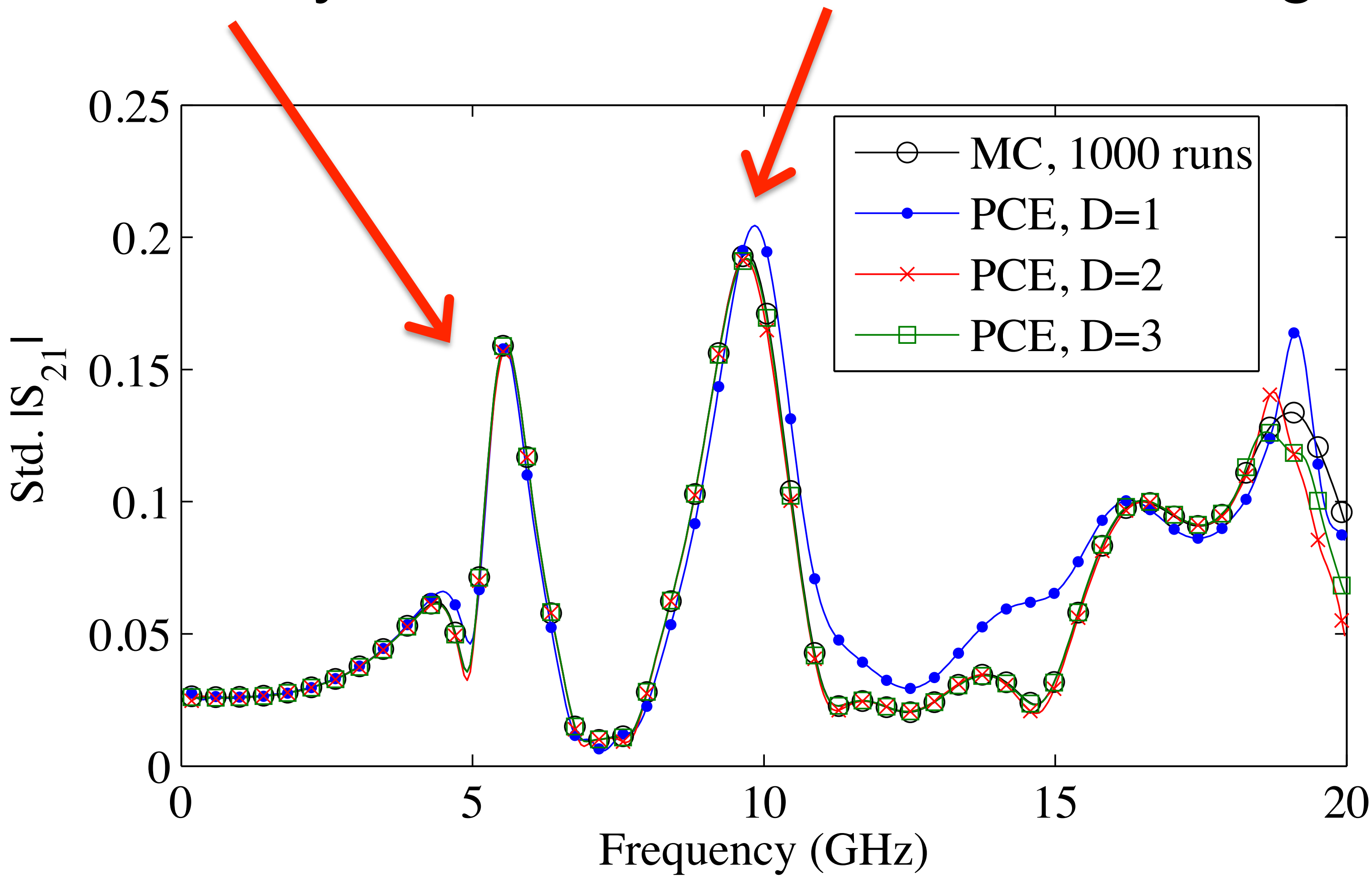
Lowpass Filter: Mean S_{21}

- Mean is well predicted over 0−20 GHz



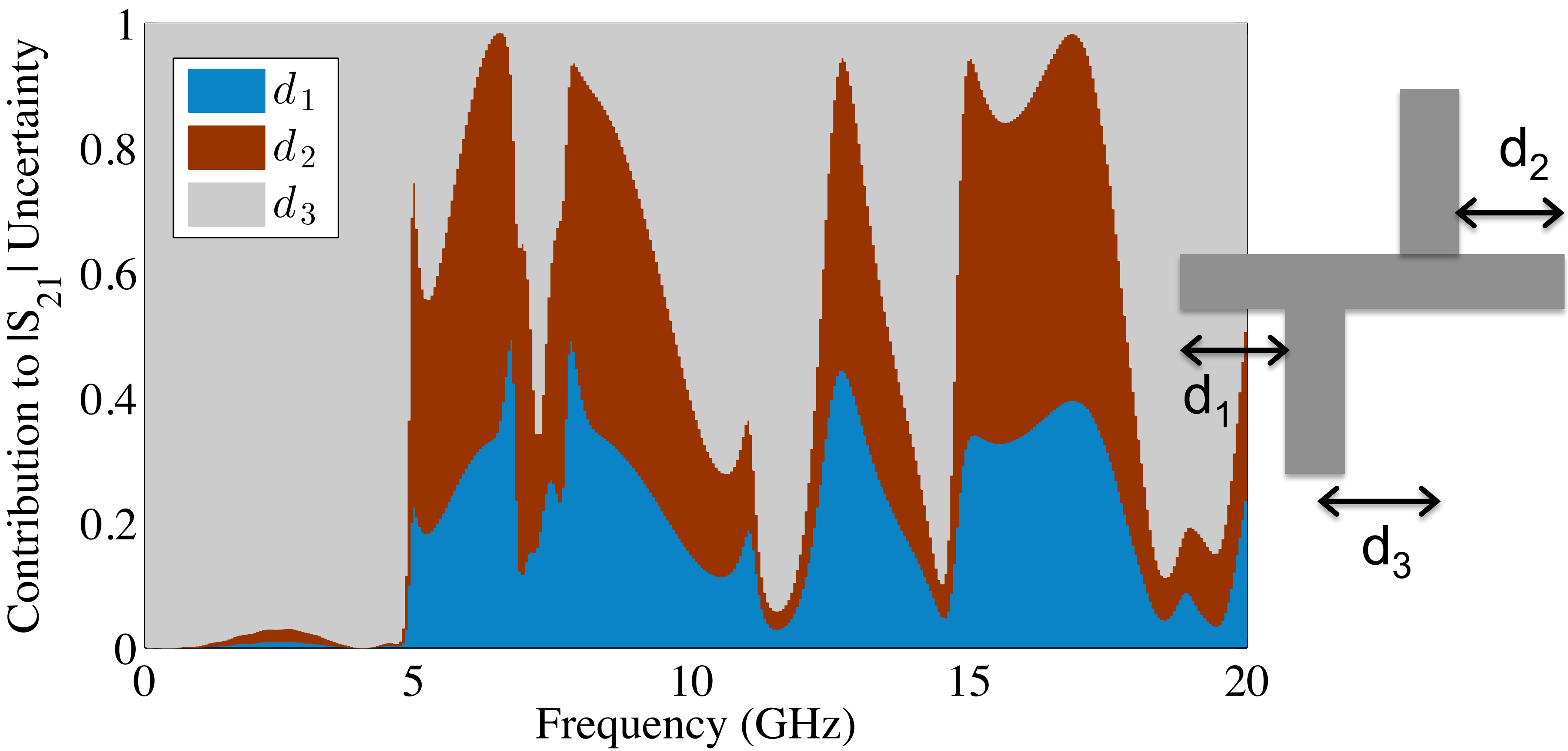
Lowpass Filter: Standard Deviation

- Uncertainty increases in the transition regions



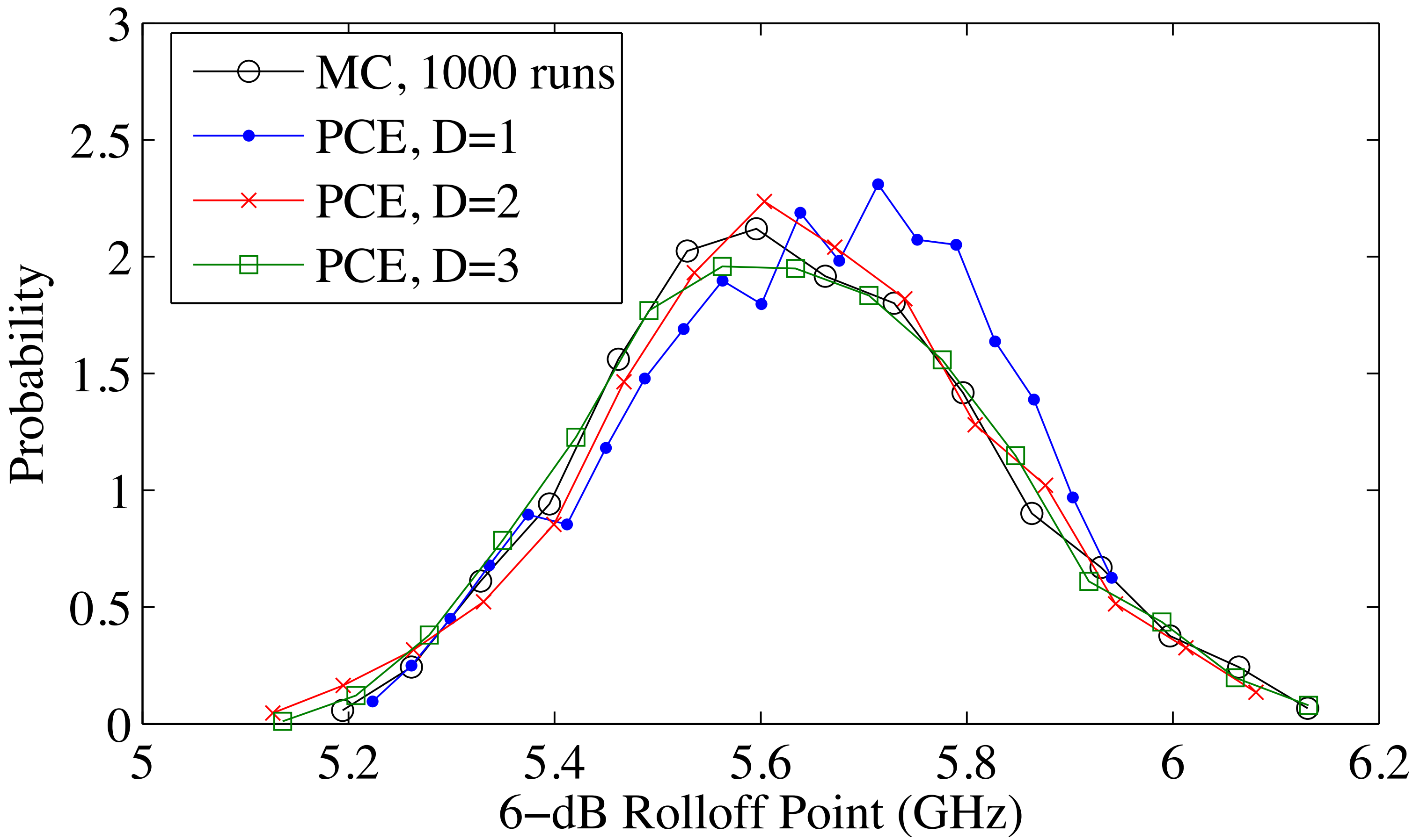
Lowpass Filter: Sensitivity Analysis

- Partial variances from PCE to estimate the contribution each parameter makes toward S_{21}



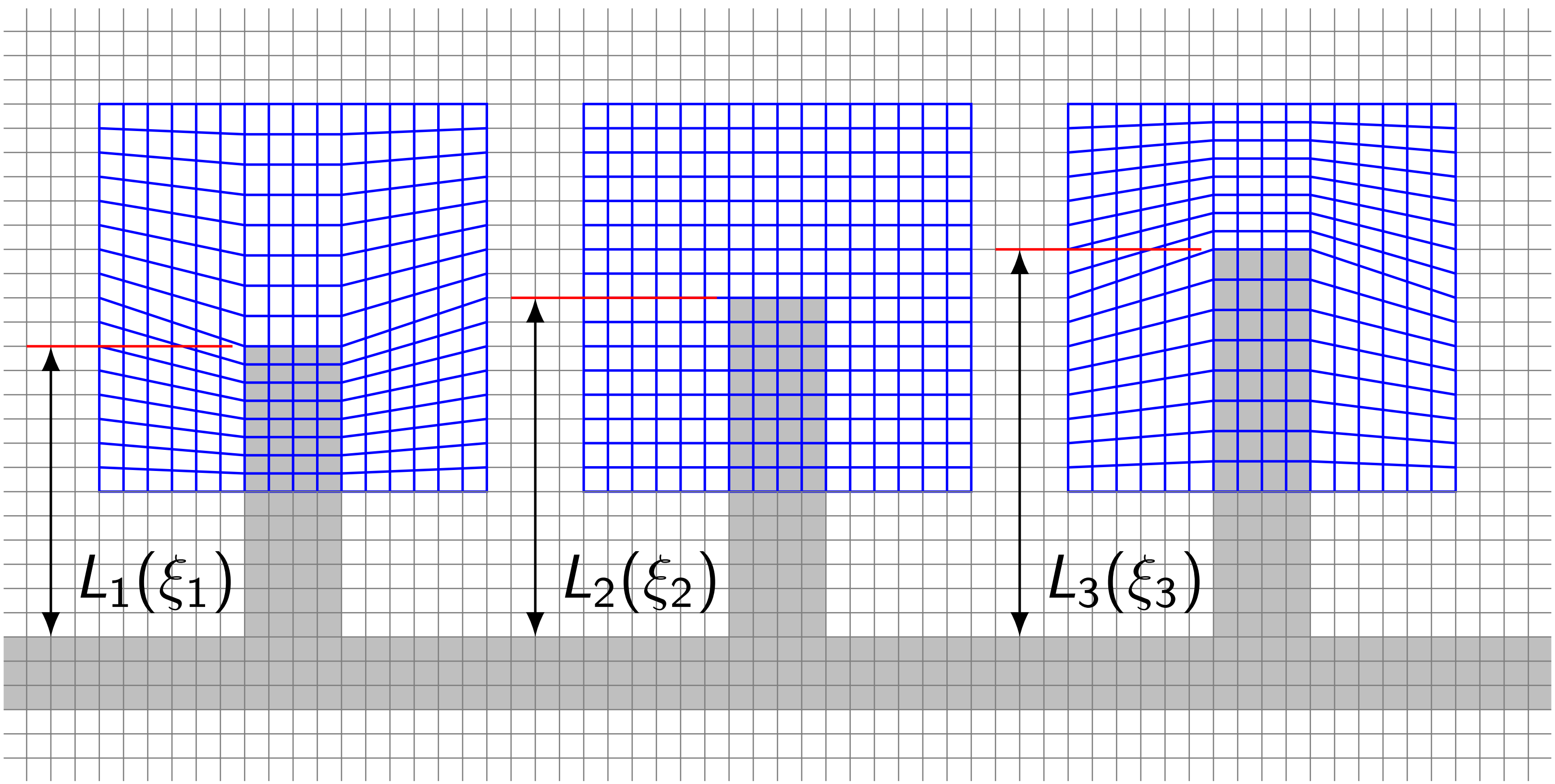
Lowpass Filter: PDF

- Convergence to MC as we increase order



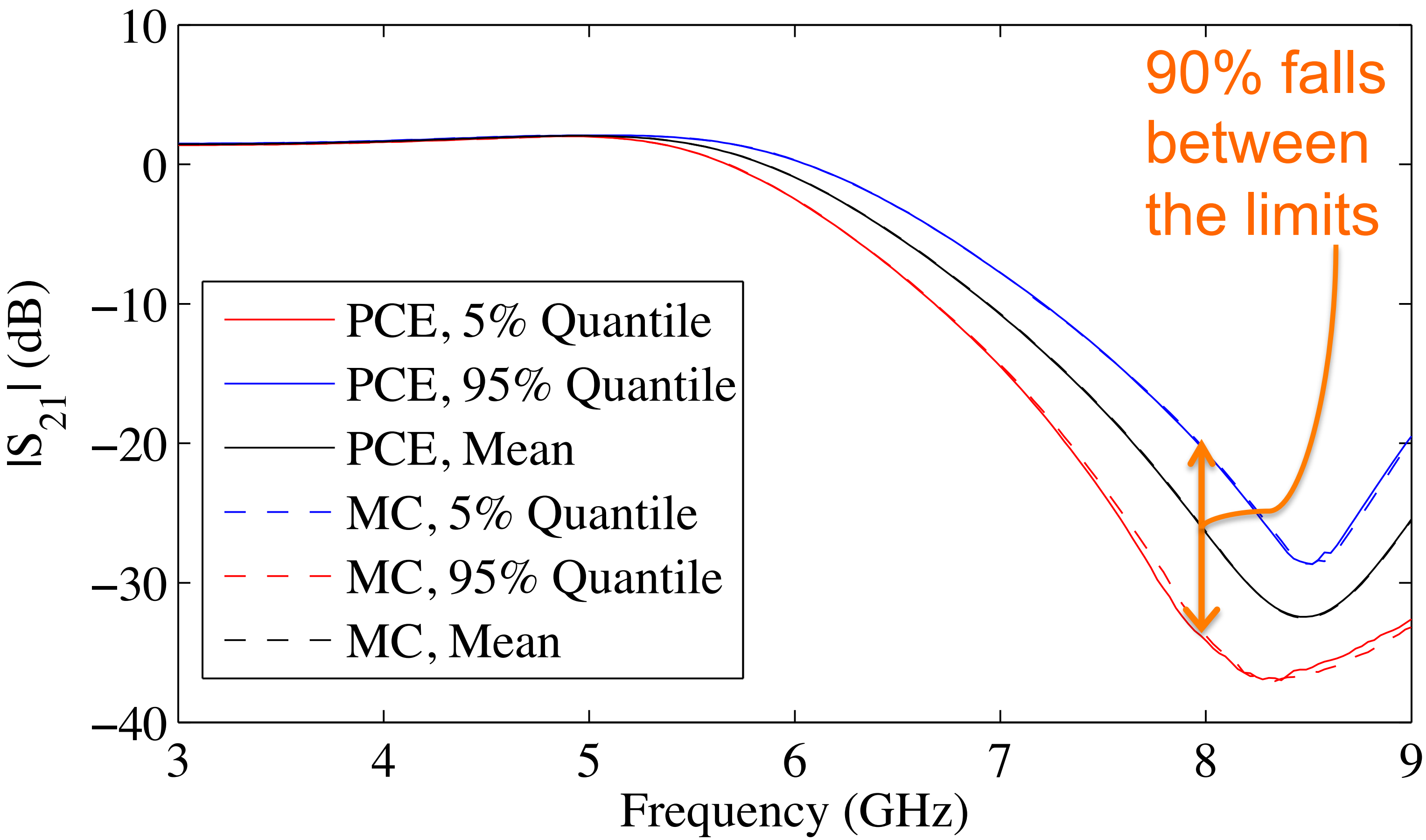
Cascaded Stubline Filter

- Aligned geometry: curvilinear mesh PCE-FDTD
- Uncertainties: L ~ β (μ = 12.5 mm, σ = 0.17 mm)



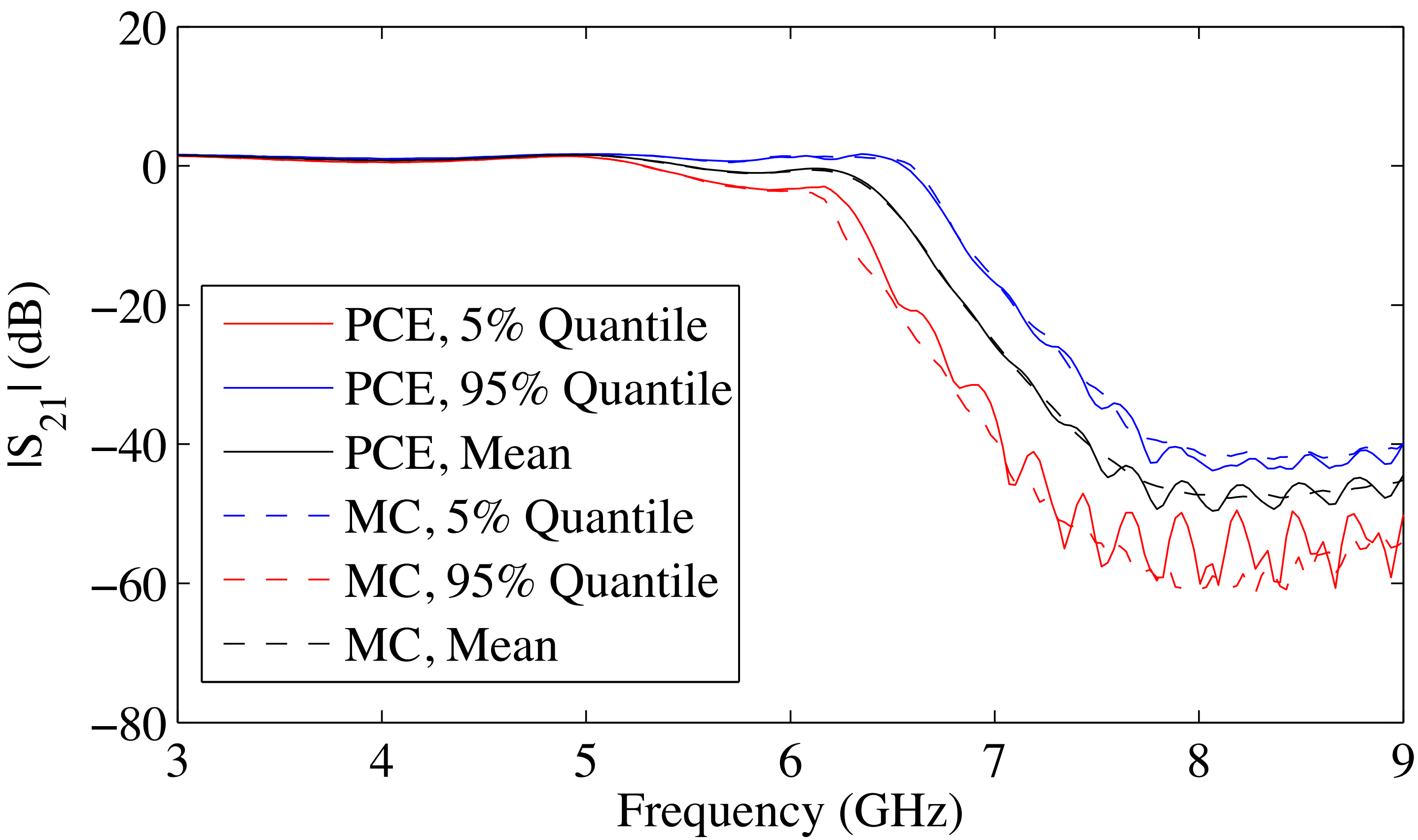
Two Stub Filter

- Uncertainty increases in the roll-off



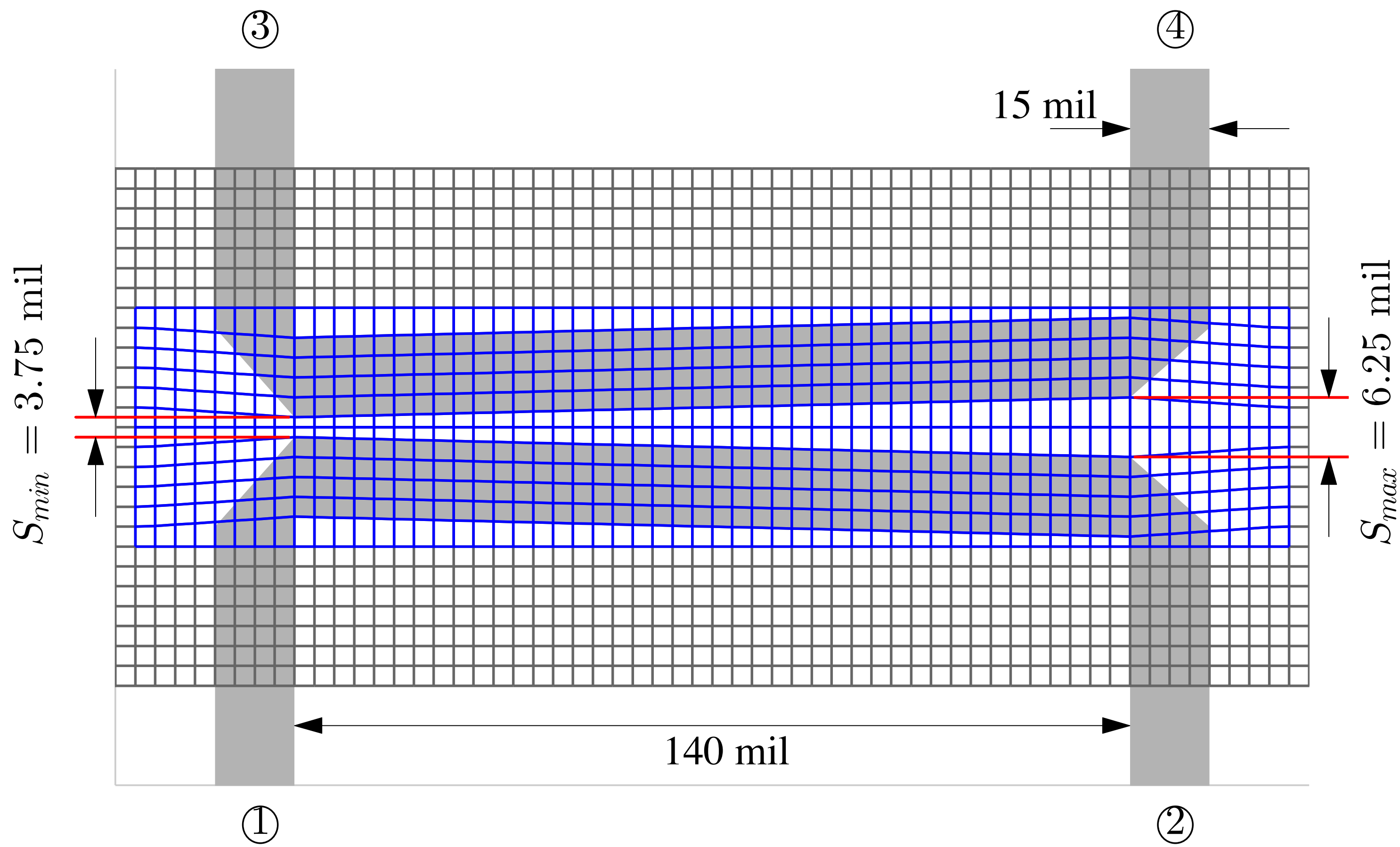
Four Stub Filter

- Uncertainty accumulates as we cascade more elements: higher orders needed to converge



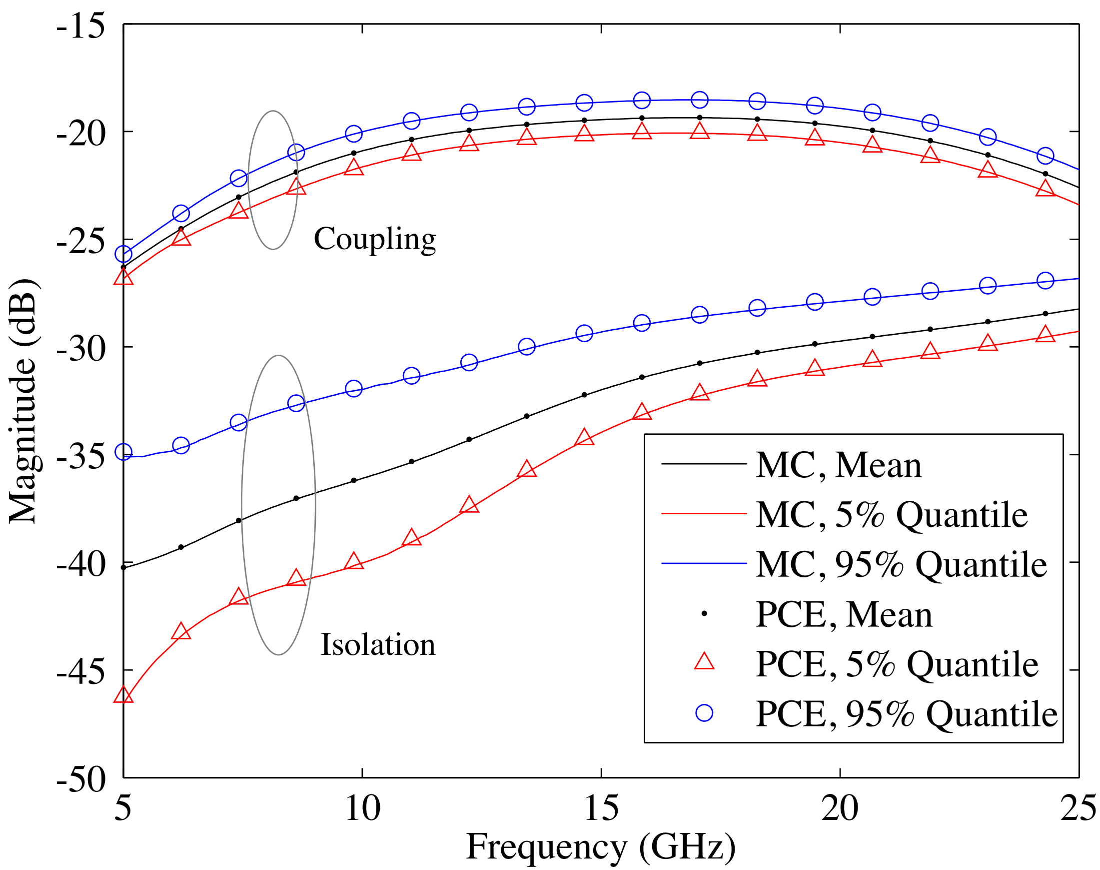
20dB Microstrip Directional Coupler

- Curvilinear mesh to introduce uncertainty in separation and orientation between the lines



Directional Coupler

- Uncertainty in coupling and isolation predicted by the PCE compare well with MC results



Relative Computational Costs

- A large number of Monte Carlo trials must be run to ensure convergence
  - 1000 trials used: multiple days
- # of PCE terms grow rapidly: $P+1=\dfrac{(N+D)!}{N!D!}$

	Monte Carlo (each trial)	Order, D	PCE-FDTD Time	$\times$ RAM
Microstrip Filter, $N=3$	4 mins	3	58 mins	$\times 20$
Cascaded Filter, $N=4$	25 mins	4	12 hours	$\times 70$
Directional Coupler, $N=2$	11 mins	4	4.5 hours	$\times 15$

Conclusions

- Novel method to characterize geometrical uncertainties in the FDTD method
- Polynomial Chaos used to expand uncertainty fields in terms of the mesh distortion
- Applied to several microwave circuits: good agreement with Monte Carlo at significantly lower computational cost

Weiner-Askey Scheme

- Relate basis functions to the PDFs of the inputs: optimal convergence[1]

Random Variable, ξ	Weiner-Askey Polynomial, Ψ	Support
Gaussian	Hermite	$[-\infty, \infty]$
Uniform	Legendre	$[a, b]$
Beta	Jacobi	$[a, b]$
Gamma	Laguerre	$[0, \infty]$

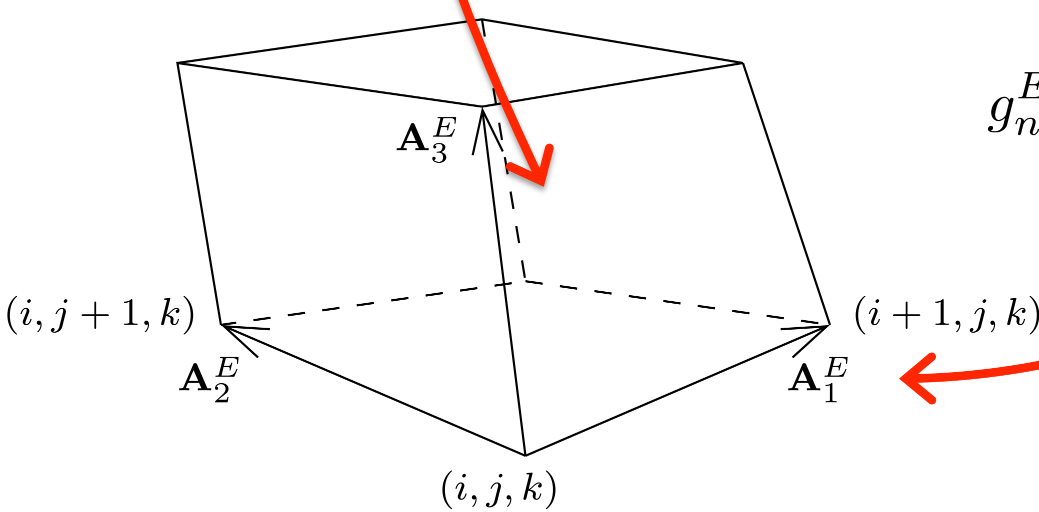
[1]D. Xiu and Karniadakis, SIAM J. Sci. Comput., **24**(2) 2002

Random Curvilinear Mesh

- Randomness in the mesh parameters introduces uncertainty in the time-domain EM fields

$$V\Big|_{i,j,k}^{E^1} = 0.25\mathbf{A}_1^E\Big|_{i,j,k} \cdot \left[\left(\mathbf{A}_2^H\Big|_{i+1,j,k} + \mathbf{A}_2^H\Big|_{i+1,j,k+1}\right) \times \left(\mathbf{A}_3^H\Big|_{i+1,j,k} + \mathbf{A}_3^H\Big|_{i+1,j+1,k}\right)\right]$$

$$g_{n,m}^E\Big|_{i,j,k} = \mathbf{A}_n^E\Big|_{i,j,k} \cdot \mathbf{A}_m^E\Big|_{i,j,k}$$



Estimating Statistics from the PCE

- Outputs are expressed as an expansion in the PC basis:

$$R(\xi) = \sum_{a=0}^{P} r_a \Psi_a(\xi)$$

- Can be shown: $\mu\left[R(\xi)\right] = r_0$

$$\sigma^2\left[R(\xi)\right] = \sum_{a=1}^{P} r_a^2 \left\langle \Psi_a^2 \right\rangle$$

- Relative sensitivities estimated by considering partial variances[1]

[1] T. Crestaux et al. Reliab. Eng. Syst. Safe., 94(7) pp. 1161-1172, 2009