Efficient Field Reconstruction Using Compressive Sensing

Andrew C. M. Austin and Michael J. Neve

***Abstract*—Compressive sensing is used to reconstruct the time-harmonic electric field created by multipath fading from a limited number of measurement points over a planar region. The multipath fading signal is shown to be the superposition of multiple plane wave components and thus has a sparse representation in the spatial-frequency domain. The discrete Fourier-basis used as the dictionary for orthogonal matching pursuit is oversampled to ensure sufficient resolution in the spatial-frequency domain. Experimental results at 10 GHz using an arbitrary plane wave expansion shows the signal-to-error ratio of the compressive sensing reconstruction is approximately 16 dB when randomly selecting only 7.5% of the total points.**

***Index Terms*—Channel Modelling, Compressive Sensing, Radiowave Propagation**

I. INTRODUCTION

The spatial patterns created by the superposition of various time-harmonic electric and magnetic fields can provide significant insights into radio-wave propagation processes, leading to the identification of the dominant propagation mechanisms for indoor and outdoor environments [1], [2]. This information can be useful to wireless system planners, for example, Poynting-vector streamlines projected through a multi-storey building can be used to infer the propagation paths and the optimal locations of frequency selective shielding to reduce unwanted interference [3], [4]. Computational electromagnetic techniques, such as the finite-difference time-domain (FDTD) or ray-tracing methods based on geometrical optics, are increasingly used to model radio wave propagation for indoor and outdoor channels and can provide accurate results. However, computational solutions can be sensitive to uncertainties in the description of the environment and in material properties [5], [6], potentially leading to misidentification of the propagation mechanisms and paths. Accordingly, *conclusive* identification of the dominant propagation mechanisms is only possible via experimental measurements of the channel in the region(s) of interest.

Previous research has focused on densely sampling the electric field over a planar or volumetric region of space to create a synthetic aperture to estimate the dominant propagation paths, e.g., [7]–[9]. However, these measurements can be time consuming to perform, as a sufficiently fine sampling grid over a region multiple wavelengths in size is typically required. For example, in [9], 10,000 spatial measurements (over a 1 m^2 grid) were used to estimate the three-dimensional angle-of-arrival for indoor ultra-wideband channels (with a sampling density of 2.8–9.6 samples/wavelength). Similarly, in [8] the sampling resolution was 6.75 samples/wavelength. Furthermore, it is often assumed that the channel remains static while the measurements are completed, but, given the (potentially) long duration required, this assumption may be invalid. A different approach was taken in [1], where to alleviate the need to densely sample the spatial field, a quasi two-dimensional conjoint cylindrical wave expansion was used to interpolate and reconstruct the interior electric field from perimeter measurements around a 0.83×0.83 m planar region.

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Contributions: This paper investigates the use of compressive sensing to reconstruct the spatial electric field distribution over a two-dimensional plane from a limited number of randomly positioned sample points. The resulting image of the radio channel can be used to identify and examine the propagation mechanisms that dominate in the measurement region. Compressive sensing is a recent signal processing technique that can very efficiently reconstruct an under-sampled signal by exploiting sparsity in one or more of the signal domains [10]. It has previously been applied to a number of problems in electromagnetics, including antenna array synthesis, inverse scattering, direction-of-arrival estimation, and imaging [11]. In [12] a phaseless compressive sensing approach was developed for near-field antenna measurements using a discrete Fourier basis. Of particular relevance to this work, in [13] compressive sensing was used in an attempt to reduce the number of measurements points required to reconstruct the multipath fading envelope for an outdoor/urban radio propagation channel. However, analysis was restricted to linear one-dimensional measurements and the sparsity model lacked an electromagnetic basis. In this paper, compressive sensing is applied to samples of the (complex) time harmonic electric field, E, not to the fading envelope, $|E|$, as in [13]. In particular, it is shown that the time harmonic field can be decomposed via the plane wave angular spectrum to a sparse representation in the spatial-frequency domain, which does not necessarily hold for $|E|$.

II. FIELD RECONSTRUCTION USING COMPRESSIVE SENSING

Many signals appear random and distinctly non-sparse in the measured domain (e.g., position), however, there often exists a sparse representation in some other domain (e.g., the spatial frequency domain). For indoor and outdoor radio channels, the spatially varying electric field measured over a planar region is caused by the superposition of multiple components, e.g., from reflection, diffraction and scattering from objects in the surrounding environment [14, pp. 217–226]. In many cases it is possible to model these propagation effects (and the resulting electric field distribution) using a superposition of plane waves [1], [14]. In this section we will briefly outline the theory of compressive sensing and develop a formulation that can be applied to reconstruct the spatial-domain electric field arising from the summation of arbitrary plane waves using a limited number of sample points.

A. Overview of Compressive Sensing

Consider a signal vector, $\mathbf{y} \in \mathbb{C}^N$, that is K-sparse with $K \ll N$, i.e., only K elements in $\mathbf{y}$ have non-zero values. Suppose also that $\mathbf{y}$ cannot be measured directly, and can only be measured in some transformed domain. The signal in the transformed domain, $\mathbf{x} \in \mathbb{C}^D$, can thus be expressed as

$$\mathbf{x} = \Psi \mathbf{y} \tag{1}$$

where Ψ is the $D \times N$ transformation matrix (typically $D = N$). It should be noted that in the transformed domain there is no *a priori* guarantee that $\mathbf{x}$ will also be sparse. Let Φ represent the $M \times D$ sampling/measurement matrix, where M represents the number of samples taken of $\mathbf{x}$. We thus have,

$$\mathbf{b} = \Phi \Psi \mathbf{y} \tag{2}$$

where $\mathbf{b}$ represents the vector of measured samples. Typically, to solve the inverse of (2) using standard l_2-norm techniques (such as least squares) requires $M \geq N$, i.e., dense sampling of $\mathbf{x}$. The theory of compressive sensing allows us to collect relatively few samples of $\mathbf{x}$ in order to estimate $\mathbf{y}$, i.e., with $M \ll N$, which would typically be an ill-posed problem as (2) would be severely under-determined. In particular, it has been shown in [10] that under conditions of sparsity $\mathbf{y}$ is the solution to the minimisation,

$$\min \|\mathbf{y}\|_0, \text{ such that } \mathbf{b} = \Phi\Psi\mathbf{y} \tag{3}$$

where $\|\mathbf{y}\|_0$ is the l_0-norm of $\mathbf{y}$. The l_0-norm is difficult to evaluate, but recent results have shown a relaxation to the l_1-norm can provide a good approximation to (3), if columns of matrix $\Phi\Psi$ are as orthogonal as possible (for all sets of columns) [10]. In practise this condition can be achieved by randomising the sampling matrix, Φ. In this paper, we solve the l_1-norm relaxation of (3) using an implementation of the orthogonal matching pursuit (OMP) algorithm. OMP is a greedy algorithm, that iteratively finds the set of components from the overcomplete dictionary, $\Phi\Psi$, that best approximates the sampled signal, $\mathbf{b}$ [15]. On each iteration, the component that maximises the inner product of the residual (initially this is $\mathbf{b}$) with $\Phi\Psi$ is selected [15]. This component is then subtracted from the residual, and the process is repeated until convergence is reached, or a maximum number of iterations have been completed.

B. Application of Compressive Sensing to an Arbitrary Plane Wave Expansion

The electric field within the two-dimensional planar region (on the $\hat{x}\hat{y}$ plane, with $z = 0$) of interest is modelled as the summation of I incident homogeneous plane waves with random phase, magnitude, and angle of arrival. In particular, the azimuth, ϕ, is assumed to be uniformly distributed over $0 - 360^\circ$. However, the elevation, θ, is restricted to be uniformly distributed over $90^\circ \pm 16^\circ$, as previous analysis of outdoor radio channels has indicated the majority of the energy arrives only slightly above or below the horizon [16]. The E_z component of the time harmonic electric field within the planar region can be written as

$$E_z(\mathbf{r}) = \sum_{i=1}^{I} E_i e^{-j\mathbf{k}_i \cdot \mathbf{r}} \tag{4}$$

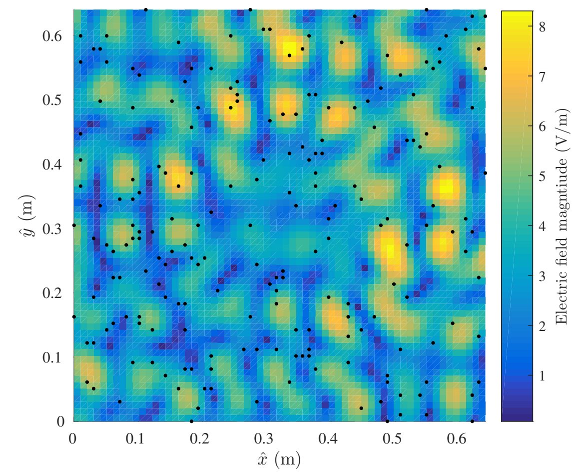
where $\mathbf{r}$ is the observation point in the two-dimensional region (i.e., the $\hat{z}$ component of $\mathbf{r}$ is zero), and $\mathbf{k}_n$ is the wave-vector of the n-th plane wave, given by

$$\mathbf{k}_i = k_0 \left[\cos(\phi_i)\sin(\theta_i)\hat{x} + \sin(\phi_i)\sin(\theta_i)\hat{y} + \cos(\phi_i)\hat{z}\right] \tag{5}$$

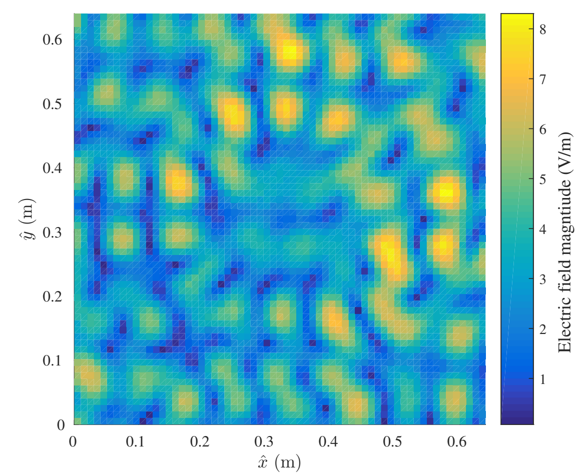
where $k_0 = \frac{2\pi}{\lambda}$ is the wave number. It is assumed that E_i is complex Gaussian distributed, i.e., the real and imaginary parts of E_i are independently Gaussian distributed.

Based on the compressive sensing framework outlined in section II-A and (4) we propose a two-dimensional inverse discrete Fourier transform (IDFT) matrix for Ψ, which can be computed as the outer product of two one-dimensional IDFT matrices. The planar E_z field is represented by $\mathbf{x}$ in (1), and the corresponding plane wave angular spectrum (via the IDFT matrix, Ψ) is represented by $\mathbf{y}$. Both $\mathbf{x}$ and $\mathbf{y}$ are inherently two-dimensional, but can be expressed as one-dimensional vectors by stacking the matrix columns. The vector of undersampled E_z field values is represented by $\mathbf{b}$ in (2), where Φ represents a random sampling/measurement matrix.

A particular challenge when using a discrete Fourier-basis in compressive sensing is spectral leakage caused by non-integer frequency components [17]. While windowing can reduce the impact of spectral leakage, it effectively reduces the size of the physical working space.



(a)



(b)

Fig. 1. (a) Magnitude of the normalised electric field created by 20 homogeneous plane waves incident on a 0.64×0.64 m planar region, also shown are 205 random sample points (indicated by $\bullet$), representing 5% of the total points; (b) Compressive sensing reconstruction of the electric field magnitude from the 205 sample points, with reconstruction SER of 22.1 dB.

In this paper we oversample the IDFT by a factor of α (i.e., $D = \alpha N$) to provide sufficient resolution in the spatial-frequency domain. The n-th column in the $D \times N$ transform matrix can thus be written as

$$\omega_n = \left[e^{j2\pi\frac{n}{N}\frac{0}{D}} \;\; e^{j2\pi\frac{n}{N}\frac{1}{D}} \;\; \ldots \;\; e^{j2\pi\frac{n}{N}\frac{d}{D}} \;\; \ldots \;\; e^{j2\pi\frac{n}{N}\frac{D-1}{D}}\right]^{\mathsf{T}} \tag{6}$$

for $n = 0 \ldots N-1$. The elevation angle is not explicitly considered in this formulation, but plane waves with $\theta_i \neq 90^\circ$ will still have a sparse representation in the plane wave angular spectrum.

III. Numerical Results

Fig. 1(a) shows the spatial distribution of $|E_z|$ across a 0.64×0.64 m planar region for $I = 20$, computed from (4). The frequency of operation is 2.45 GHz and due to constructive/destructive interference, the E_z field is detail-rich in the spatial domain, necessitating a high sampling density (in this case a spatial grid of 0.01 m is used). However, in the spatial-frequency domain, the field is sparse, in this case containing only the $I = 20$ components representing the plane wave angular spectrum. Also shown in Fig. 1(a) are the randomly

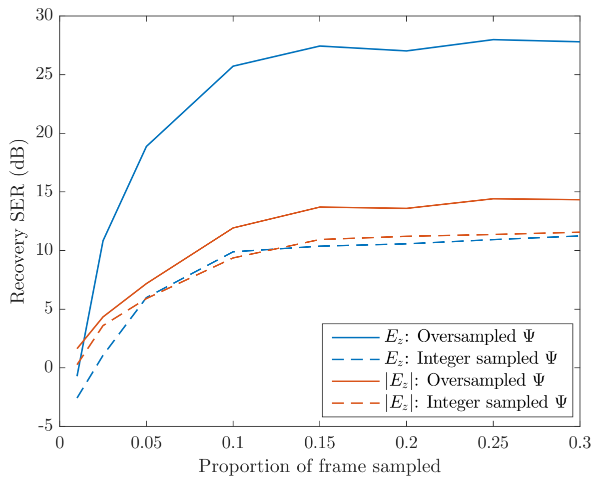


Fig. 2. SER of the reconstruction, calculated using (7) and averaged over 50 trials, as the proportion of the frame that is sampled is increased.

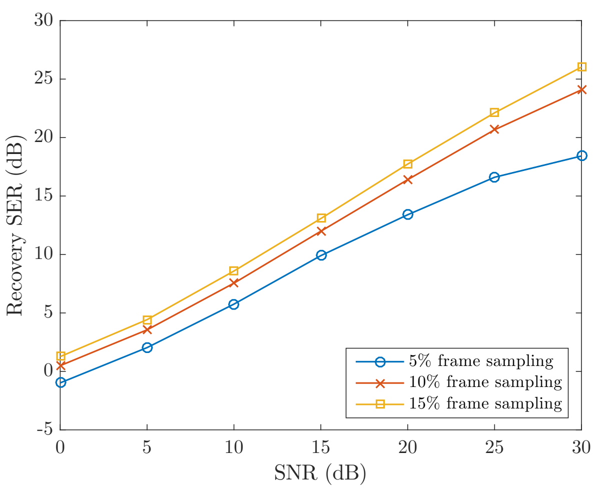
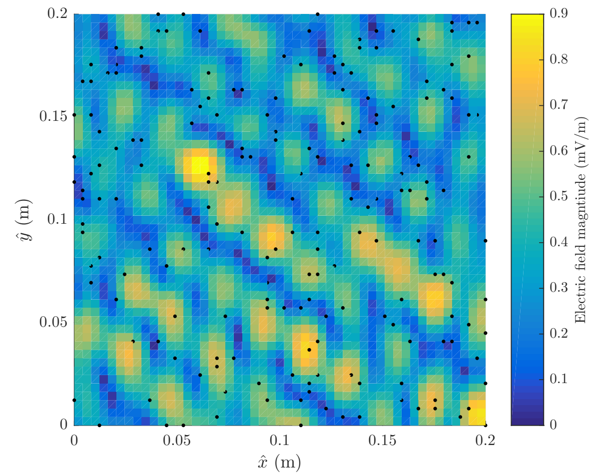
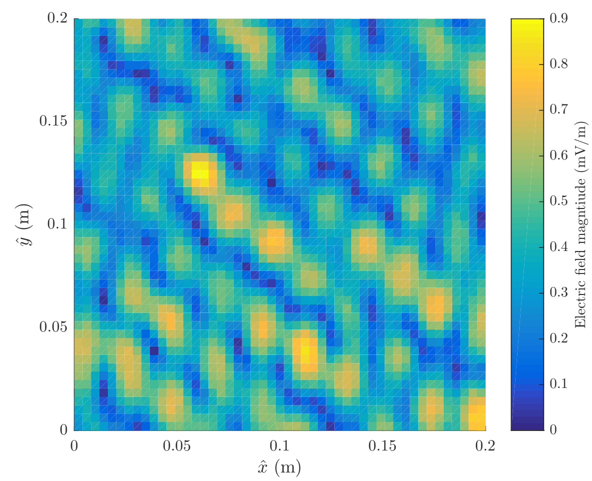


Fig. 3. Impact of random Gaussian noise on SER of the reconstruction, averaged over 50 trials, for 5%, 10% and 15% sampling.



(a)



(b)

Fig. 4. (a) Experimentally measured E field magnitude at 10 GHz over a $6.7\lambda \times 6.7\lambda$ (0.20×0.20 m) region, also shown are 188 random sample points (indicated by $\bullet$), representing 7.5% of the full data-set; (b) Compressive sensing reconstruction of the electric field from the 188 random sample points.

selected locations (uniformly and independently distributed in both $\hat{x}$ and $\hat{y}$) of the $M = 205$ sample points, representing 5% of the total number of grid points, $N = 4096$. The reconstruction of a spatial-domain signal from the spatial-frequency domain coefficients estimated via compressive sensing is plotted in Fig. 1(b). A qualitative comparison between Fig. 1(a) and (b) suggests that the compressive sensing reconstruction accurately reproduces the E_z field. The accuracy of the reconstruction is quantified by the signal-to-error ratio (SER),

$$\text{SER} = 20\log_{10}\frac{\|E_z\|_2}{\|E_z - \widehat{E}_z\|_2} \tag{7}$$

where $\widehat{E}_z$ is the estimated E_z in the spatial domain. For the result shown in Fig. 1(b) the SER is 22.1 dB.

Fig. 2 plots the SER as the number of sample points used for compressive sensing varies. For each case the SER is averaged over 50 trials. Sampling the complex E_z field yields a considerable improvement in the SER, compared to sampling the envelope of the field, $|E_z|$. The spectral-frequency domain representation of the electric field for the superposition of I plane-waves consists of I Dirac delta functions (i.e., perfectly sparse). However, for $|E_z|$ the spectral-frequency representation contains significantly more than I non-zero components. It is also observed that oversampling ($\alpha = 5$) the E_z field significantly increases the SER (by up to 15 dB), compared to the case when the fields are sampled on the integer ($\alpha = 1$) DFT grid. A smaller increase is also observed for $|E_z|$ fields. For both E_z and $|E_z|$ fields on the oversampled and integer grids, no significant increase in the SER is observed beyond approximately 10% sampling. While oversampling allows the position of the peaks in the spatial-frequency domain to be accurately determined, it does not eliminate the leakage of energy into adjacent spatial-frequency components and is responsible for this effect. Only a limited number of these leakage components can be included in the sparse representation, leading to a ceiling in the SER of the reconstruction.

To estimate the sensitivity of the compressive sensing approach to measurement errors, random (complex) Gaussian noise was added to the sampled fields calculated using (4). Fig. 3 plots the resulting SER for 5%, 10% and 15% sampling—using the oversampled approach outlined in Section II-B—as the signal-to-noise ratio (SNR) is varied. For each case the SER is averaged over 50 trials. This result shows the quality of compressive sensing reconstruction can depend significantly on the SNR of the measurements.

IV. Experimental Validation

The multipath fading pattern in a typical open-plan office/laboratory space was measured over 8–12 GHz using an xy positioner and an Agilent E8364A network analyser. An X-band horn antenna was mounted approximately 1.5 m from the floor and connected to port 1 of the network analyser via a 1.5 m cable. Similarly, a biconical antenna, at the same height and mounted on the xy positioner was connected to port 2. The antennas were separated by a distance of approximately 3 m, and to create a multipath channel the X-band horn antenna was orientated to ensure there was no line-of-sight path to the biconical antenna. 2500 measurements of the magnitude and phase of the electric field were collected across a 0.20×0.20 m region on a 50×50 grid, representing a sampling density of approximately 6–9 samples/wavelength.

Fig. 4(a) shows the magnitude of the 10 GHz electric field measured at the 2500 points over the $6.7\lambda \times 6.7\lambda$ (0.20×0.20 m) region, also indicated are the locations of 288 randomly positioned sample points, representing 7.5% of the full data set. Fig. 4(b) shows the compressed sensing reconstruction of the complex-valued electric field using the 288 samples, with a reconstruction SER of 15.9 dB.

V. Conclusions

Over a localised region, the spatial fading pattern of a radio channel can be modelled as the superposition of a finite number of plane waves with arbitrary magnitude and angles of arrival. Accordingly, the resulting electric field has a sparse representation in the spatial-frequency domain. However, sampling can only be performed in the spatial domain, where, due to multipath, the strength of the field can vary significantly over a fraction of a wavelength. This in turn requires a high sampling density, which is is often time-consuming and costly. Compressive sensing allows us to exploit the sparsity in the spatial-frequency domain to significantly reduce the number of samples that must be collected in order to accurately reconstruct the full field. Experimental results at 10 GHz results show that by sampling only 7.5% of the complex E_z field over a $6.7\lambda \times 6.7\lambda$ planar region, a reconstruction signal-to-error ratio of approximately 16 dB can be achieved.

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